Instructional Objectives

After reading this chapter the student will be able to

1. Differentiate between various structural forms such as beams, plane truss, space truss, plane frame, space frame, arches, cables, plates and shells.
2. State and use conditions of static equilibrium.
3. Calculate the degree of static and kinematic indeterminacy of a given structure such as beams, truss and frames.
4. Differentiate between stable and unstable structure.
5. Define flexibility and stiffness coefficients.
6. Write force-displacement relations for simple structure.

1.1 Introduction

Structural analysis and design is a very old art and is known to human beings since early civilizations. The Pyramids constructed by Egyptians around 2000 B.C. stands today as the testimony to the skills of master builders of that civilization. Many early civilizations produced great builders, skilled craftsmen who constructed magnificent buildings such as the Parthenon at Athens (2500 years old), the great Stupa at Sanchi (2000 years old), Taj Mahal (350 years old), Eiffel Tower (120 years old) and many more buildings around the world. These monuments tell us about the great feats accomplished by these craftsmen in analysis, design and construction of large structures. Today we see around us countless houses, bridges, fly-overs, high-rise buildings and spacious shopping malls. Planning, analysis and construction of these buildings is a science by itself. The main purpose of any structure is to support the loads coming on it by properly transferring them to the foundation. Even animals and trees could be treated as structures. Indeed biomechanics is a branch of mechanics, which concerns with the working of skeleton and muscular structures. In the early periods houses were constructed along the riverbanks using the locally available material. They were designed to withstand rain and moderate wind. Today structures are designed to withstand earthquakes, tsunamis, cyclones and blast loadings. Aircraft structures are designed for more complex aerodynamic loadings. These have been made possible with the advances in structural engineering and a revolution in electronic computation in the past 50 years. The construction material industry has also undergone a revolution in the last four decades resulting in new materials having more strength and stiffness than the traditional construction material.

In this book we are mainly concerned with the analysis of framed structures (*beam, plane truss, space truss, plane frame, space frame and grid*), arches, cables and suspension bridges subjected to static loads only. The methods that we would be presenting in this course for analysis of structure were developed based on certain energy principles, which would be discussed in the first module.

1.2 Classification of Structures

All structural forms used for load transfer from one point to another are 3-dimensional in nature. In principle one could model them as 3-dimensional elastic structure and obtain solutions (response of structures to loads) by solving the associated partial differential equations. In due course of time, you will appreciate the difficulty associated with the 3-dimensional analysis. Also, in many of the structures, one or two dimensions are smaller than other dimensions. This geometrical feature can be exploited from the analysis point of view. The dimensional reduction will greatly reduce the complexity of associated governing equations from 3 to 2 or even to one dimension. This is indeed at a cost. This reduction is achieved by making certain assumptions (like Bernoulli-Euler' kinematic assumption in the case of beam theory) based on its observed behaviour under loads. Structures may be classified as 3-, 2- and 1-dimensional (see Fig. 1.1(a) and (b)). This simplification will yield results of reasonable and acceptable accuracy. Most commonly used structural forms for load transfer are: beams, plane truss, space truss, plane frame, space frame, arches, cables, plates and shells. Each one of these structural arrangement supports load in a specific way.

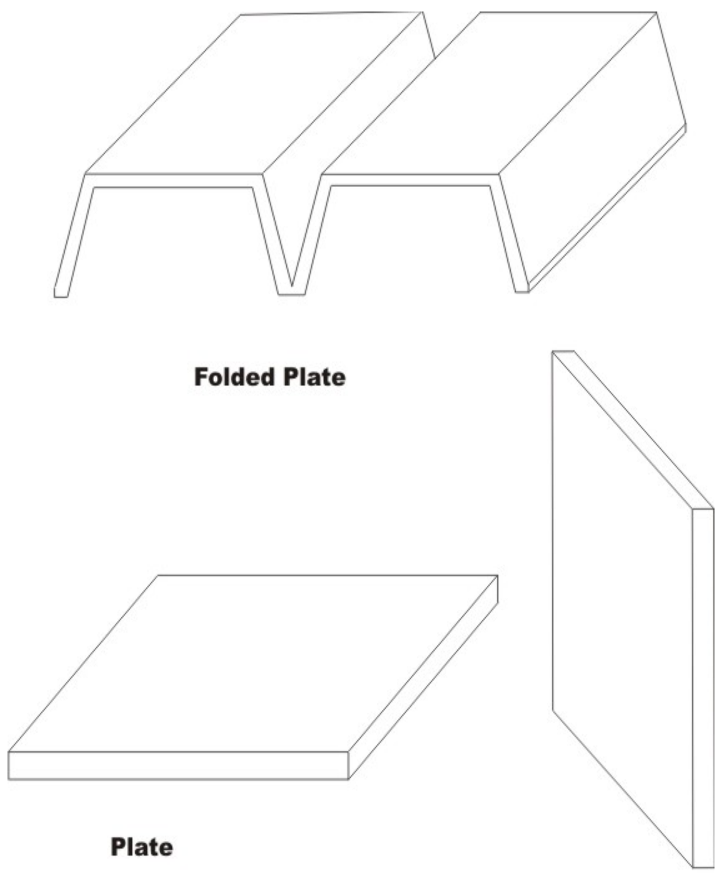


Fig 1.1(a) 2D and 3D Structural Forms

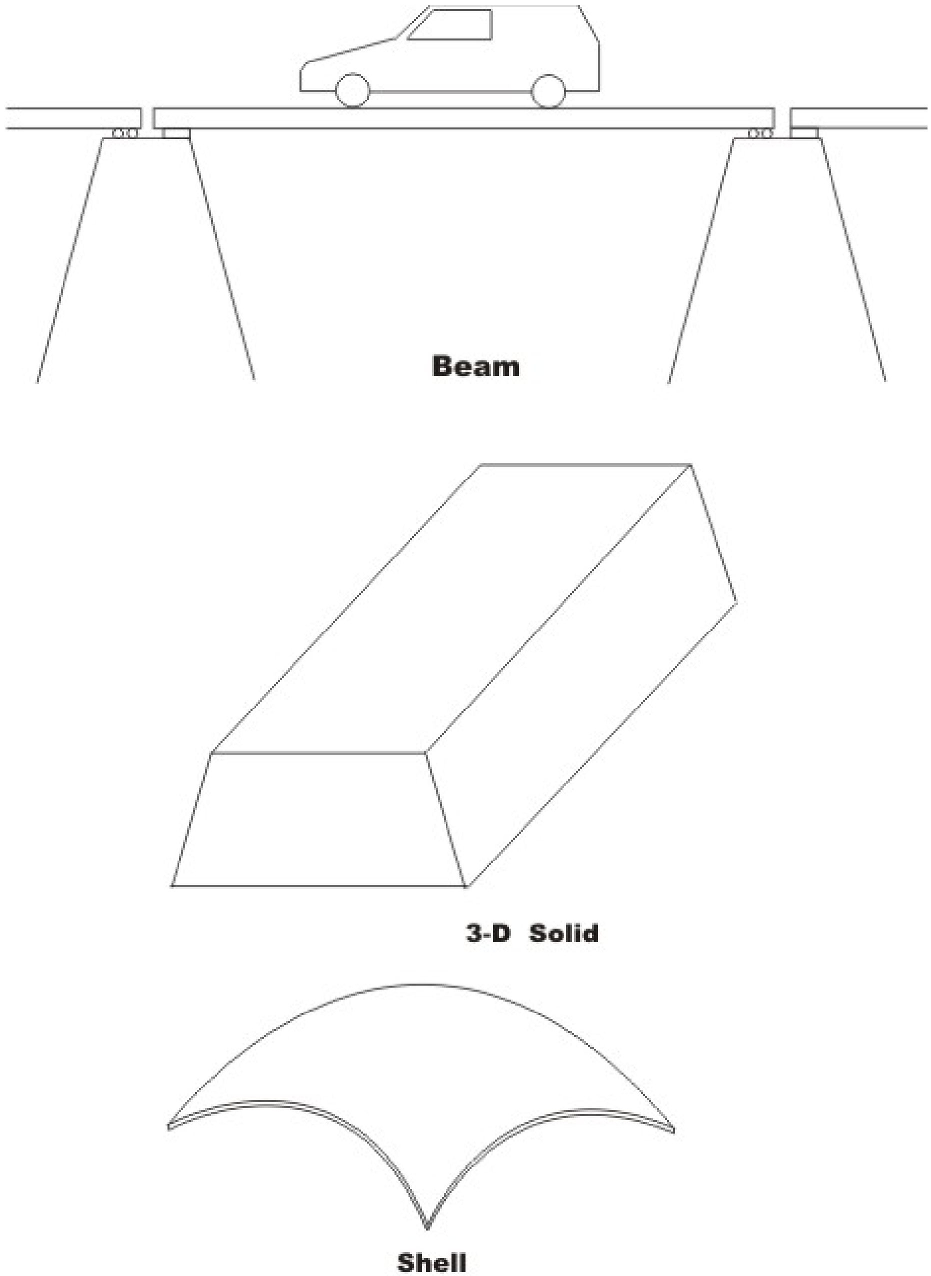
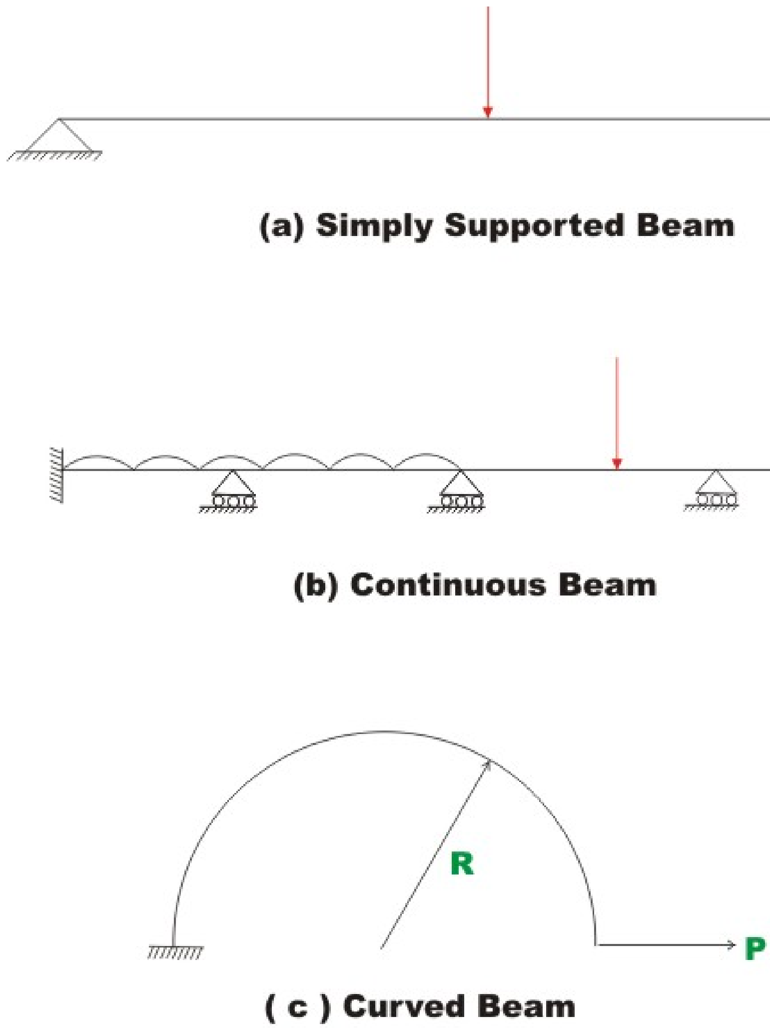


Fig 1.1(b) Commonly Used Structural Forms

Beams are the simplest structural elements that are used extensively to support loads. They may be straight or curved ones. For example, the one shown in Fig. 1.2 (a) is hinged at the left support and is supported on roller at the right end. Usually, the loads are assumed to act on the beam in a plane containing the axis of symmetry of the cross section and the beam axis. The beams may be supported on two or more supports as shown in Fig. 1.2(b). The beams may be curved in plan as shown in Fig. 1.2(c). Beams carry loads by deflecting in the same plane and it does not twist. It is possible for the beam to have no axis of symmetry. In such cases, one needs to consider unsymmetrical bending of beams. In general, the internal stresses at any cross section of the beam are: bending moment, shear force and axial force.



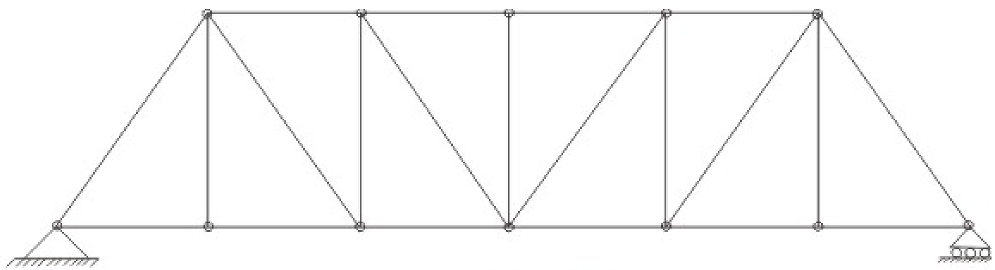
(a) Simply Supported Beam

(b) Continuous Beam

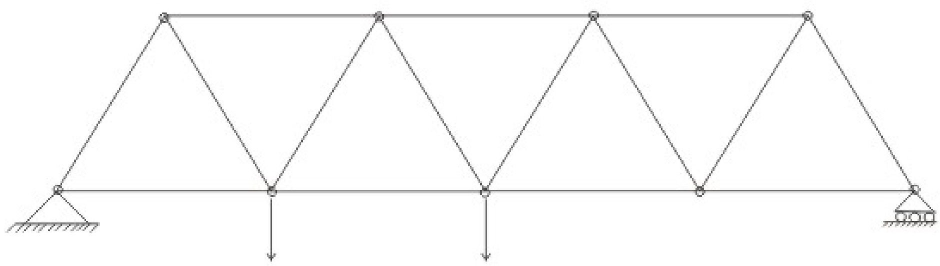
(c) Curved Beam

Fig 1.2 Beams

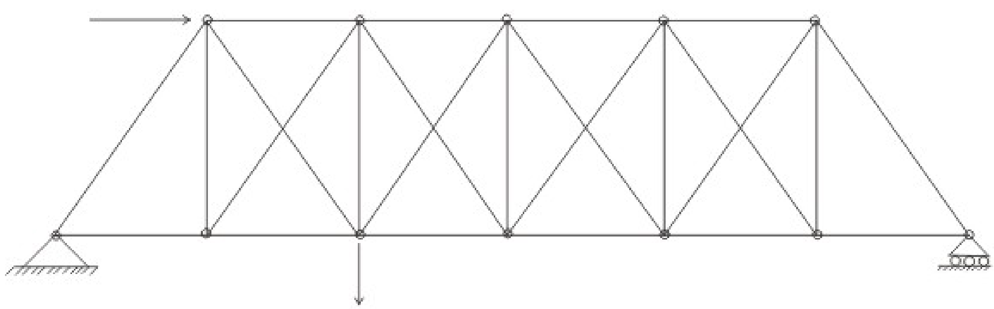
In India, one could see ***plane trusses*** (vide Fig. 1.3 (a),(b),(c)) commonly in Railway bridges, at railway stations, and factories. Plane trusses are made of short thin members interconnected at hinges into triangulated patterns. For the purpose of analysis statically equivalent loads are applied at joints. From the above definition of truss, it is clear that the members are subjected to only axial forces and they are constant along their length. Also, the truss can have only hinged and roller supports. In field, usually joints are constructed as rigid by welding. However, analyses were carried out as though they were pinned. This is justified as the bending moments introduced due to joint rigidity in trusses are negligible. Truss joint could move either horizontally or vertically or combination of them. In ***space truss*** (Fig. 1.3 (d)), members may be oriented in any direction. However, members are subjected to only tensile or compressive stresses. Crane is an example of space truss.



(a) Pratt Truss

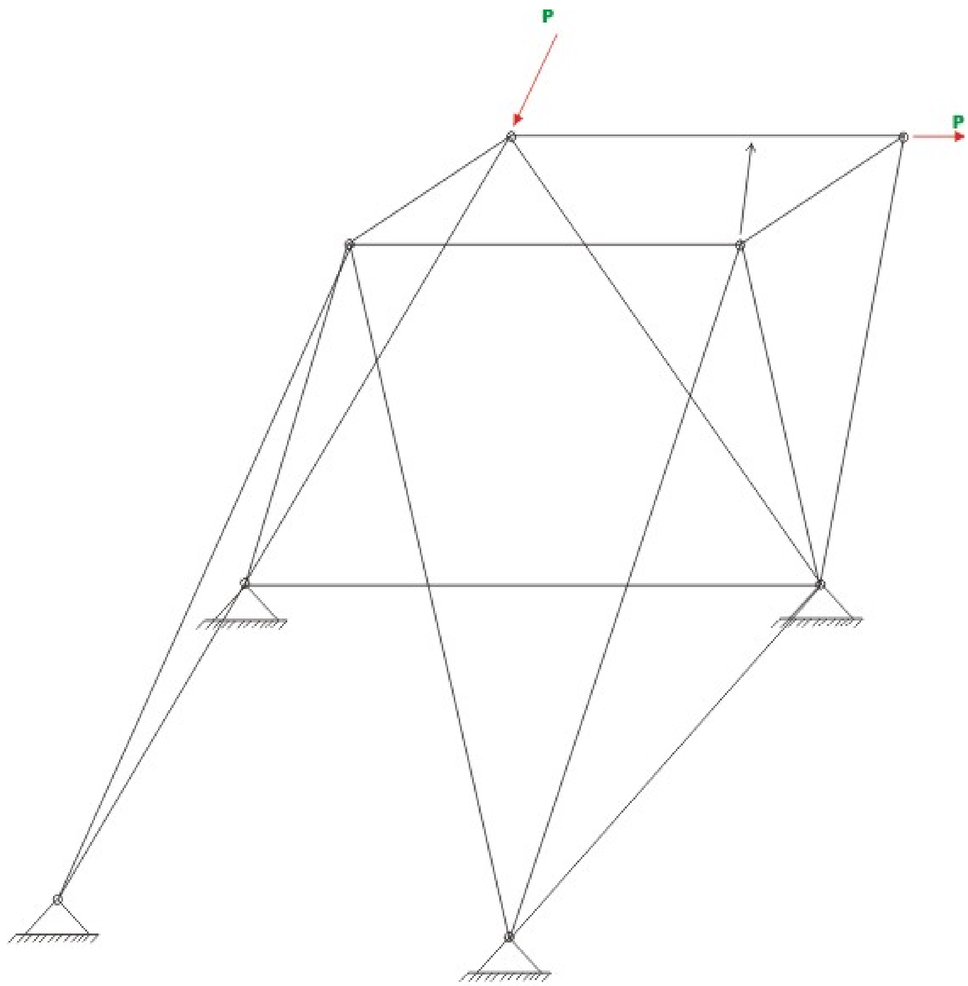


(b) Warren Truss



(c) Double Warren Truss

Fig 1.3 Trusses



(d) Space Truss

Plane frames are also made up of beams and columns, the only difference being they are rigidly connected at the joints as shown in the Fig. 1.4 (a). Major portion of this course is devoted to evaluation of forces in frames for variety of loading conditions. Internal forces at any cross section of the plane frame member are: bending moment, shear force and axial force. As against plane frame, ***space frames*** (vide Fig. 1.4 (b)) members may be oriented in any direction. In this case, there is no restriction of how loads are applied on the space frame.

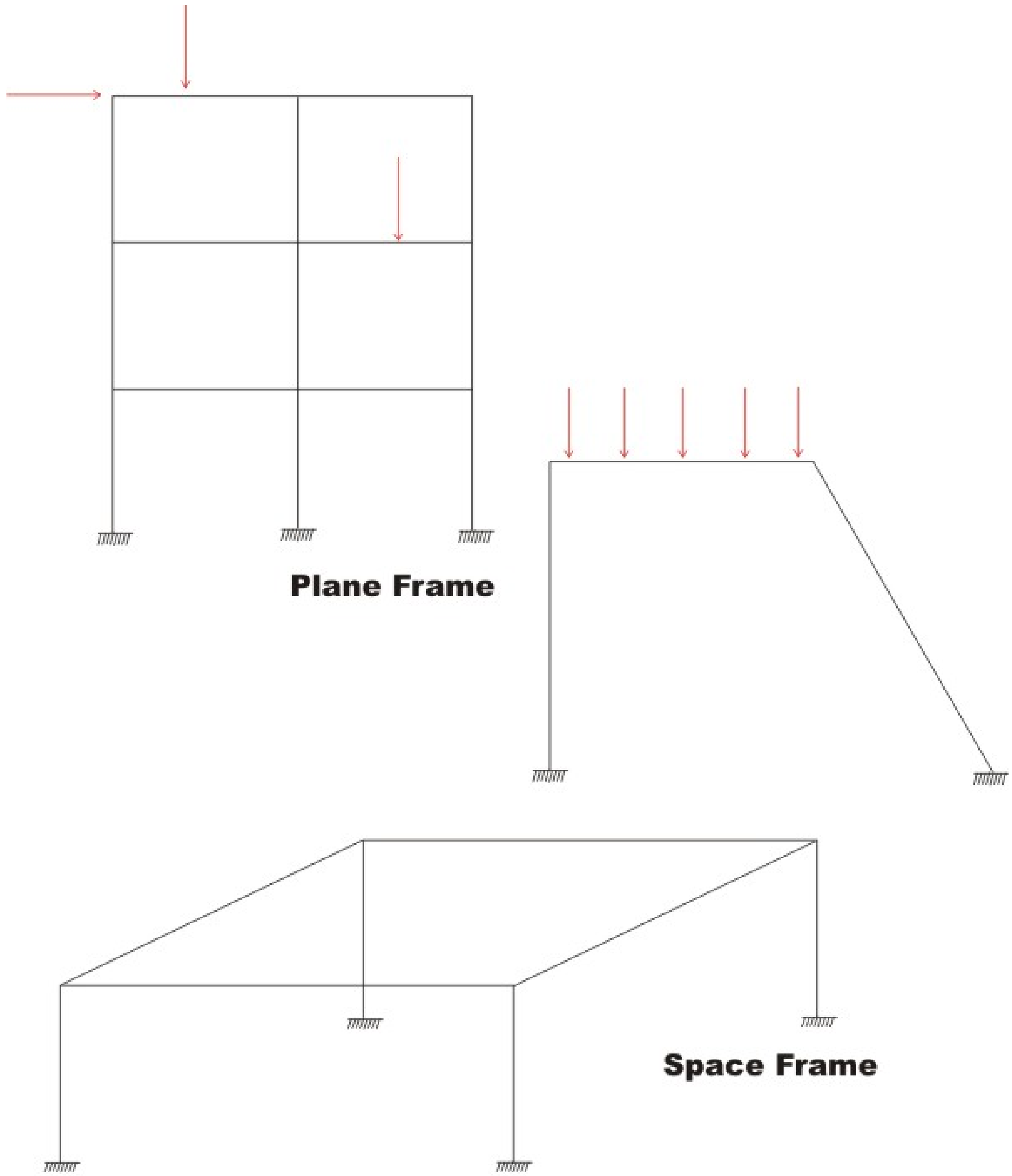


Fig 1.4 Frames

1.3 Equations of Static Equilibrium

Consider a case where a book is lying on a frictionless table surface. Now, if we apply a force F_1 horizontally as shown in the Fig.1.5 (a), then it starts moving in the direction of the force. However, if we apply the force perpendicular to the book as in Fig. 1.5 (b), then book stays in the same position, as in this case the vector sum of all the forces acting on the book is zero. When does an object move and when does it not? This question was answered by Newton when he formulated his famous second law of motion. In a simple vector equation it may be stated as follows:

$$\sum_{i=1}^{n} F_i = ma \qquad (1.1)$$

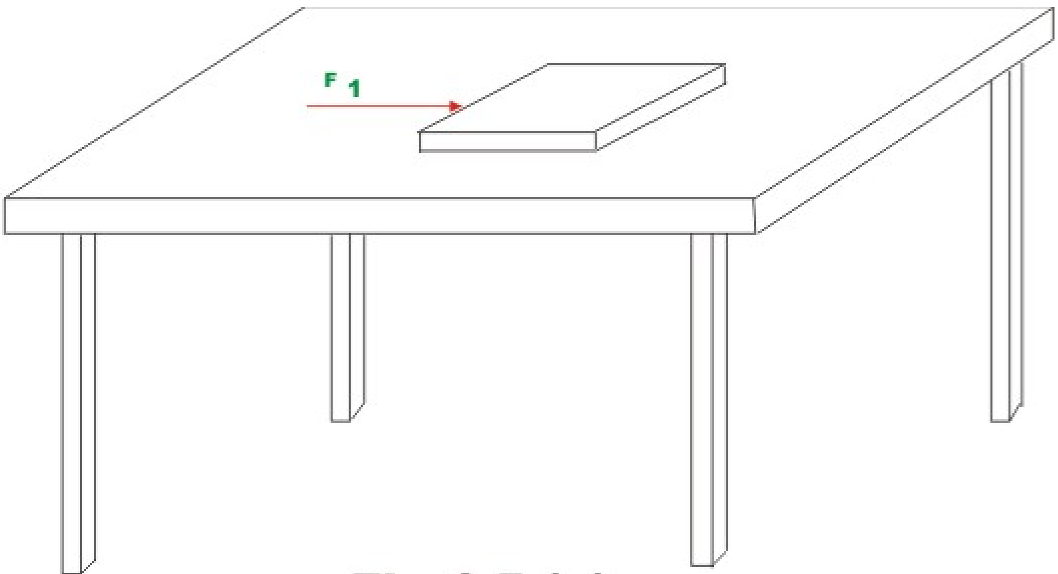


Fig 1.5 (a)

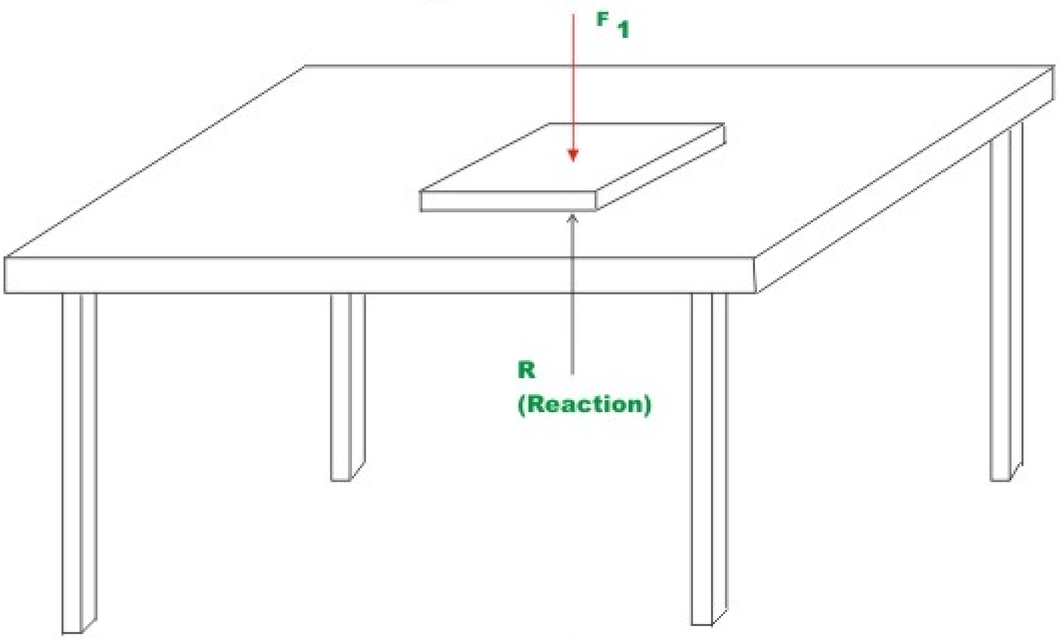


Fig 1.5(b)

where $\sum_{i=1}^{n} F_i$ is the vector sum of all the external forces acting on the body, m is the total mass of the body and a is the acceleration vector. However, if the body is in the state of static equilibrium then the right hand of equation (1.1) must be zero. Also for a body to be in equilibrium, the vector sum of all external moments ($\sum M = 0$) about an axis through any point within the body must also vanish. Hence, the book lying on the table subjected to external force as shown in Fig. 1.5 (b) is in static equilibrium. The equations of equilibrium are the direct consequences of Newton's second law of motion. A vector in 3-dimensions can be resolved into three orthogonal directions viz., *x, y* and *z* (Cartesian) co-ordinate axes. Also, if the resultant force vector is zero then its components in three mutually perpendicular directions also vanish. Hence, the above two equations may also be written in three co-ordinate axes directions as follows:

$$\sum F_x = 0 ; \ \sum F_y = 0 ; \ \sum F_z = 0 \tag{1.2a}$$

$$\sum M_x = 0 ; \sum M_y = 0 ; \sum M_z = 0 \tag{1.2b}$$

Now, consider planar structures lying in $xy-$plane. For such structures we could have forces acting only in x and y directions. Also the only external moment that could act on the structure would be the one about the z-axis. For planar structures, the resultant of all forces may be a force, a couple or both. The static equilibrium condition along x-direction requires that there is no net unbalanced force acting along that direction. For such structures we could express equilibrium equations as follows:

$$\sum F_x = 0 ; \sum F_y = 0 ; \sum M_z = 0 \tag{1.3}$$

Using the above three equations we could find out the reactions at the supports in the beam shown in Fig. 1.6. After evaluating reactions, one could evaluate internal stress resultants in the beam. Admissible or correct solution for reaction and internal stresses must satisfy the equations of static equilibrium for the entire structure. They must also satisfy equilibrium equations for any part of the structure taken as a free body. If the number of unknown reactions is more than the number of equilibrium equations (as in the case of the beam shown in Fig. 1.7), then we can not evaluate reactions with only equilibrium equations. Such structures are known as the statically indeterminate structures. In such cases we need to obtain extra equations (*compatibility equations*) in addition to equilibrium equations.

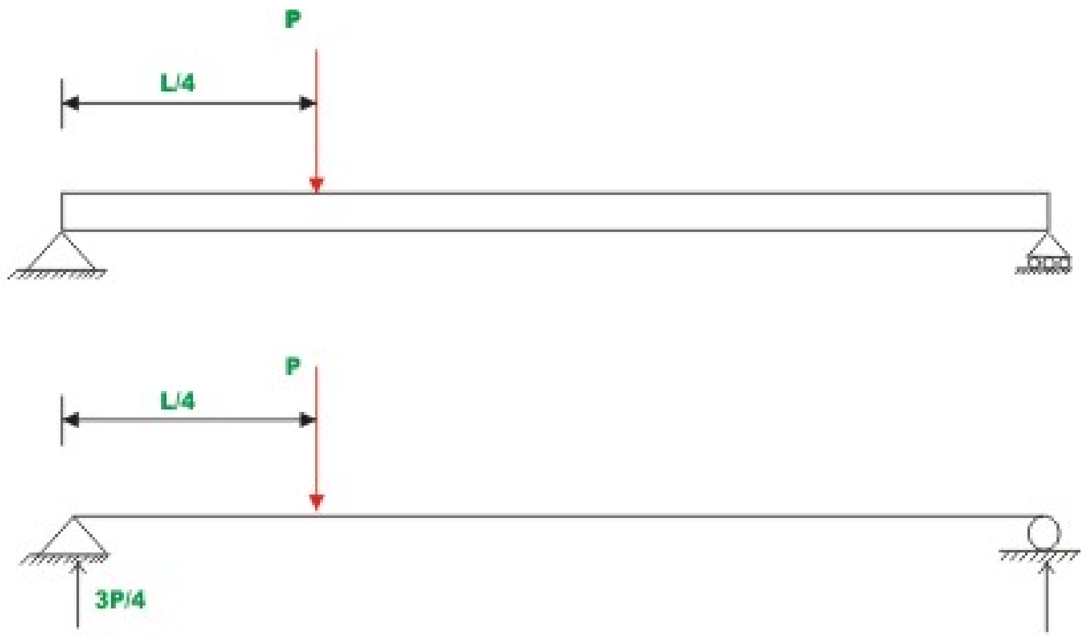


Fig 1.6 Statically Determinate Beam

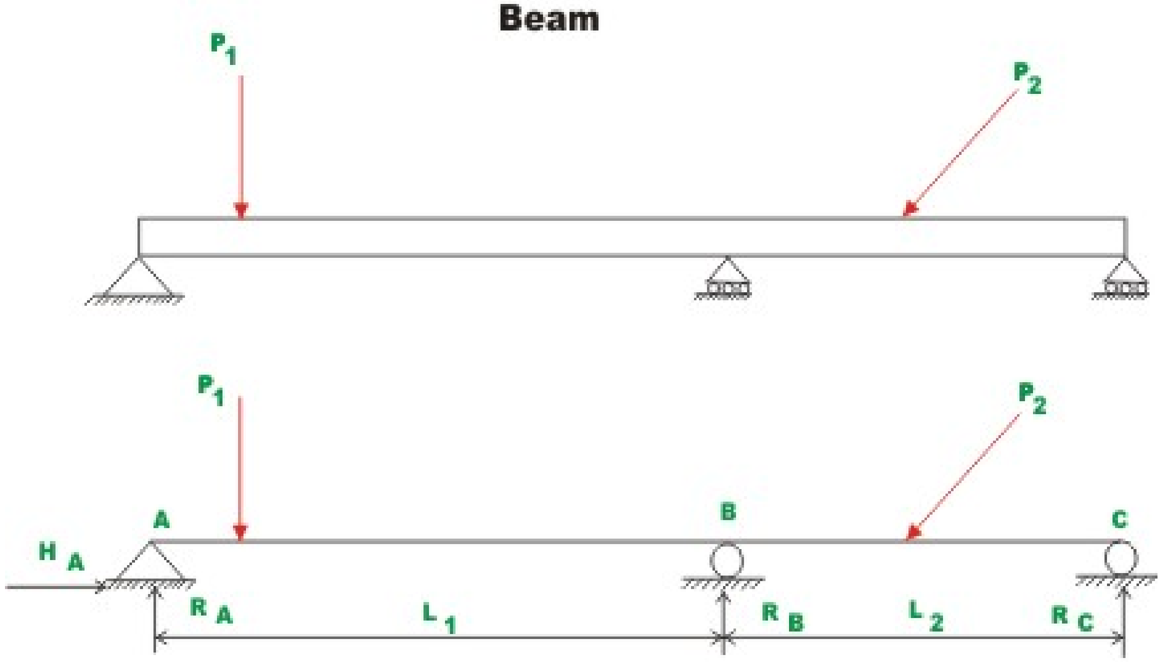


Fig 1.7 Statically Indeterminate Beam

1.4 Static Indeterminacy

The aim of structural analysis is to evaluate the external reactions, the deformed shape and internal stresses in the structure. If this can be accomplished by equations of equilibrium, then such structures are known as determinate structures. However, in many structures it is not possible to determine either reactions or internal stresses or both using equilibrium equations alone. Such structures are known as the statically indeterminate structures. The indeterminacy in a structure may be external, internal or both. A structure is said to be externally indeterminate if the number of reactions exceeds the number of equilibrium equations. Beams shown in Fig.1.8(a) and (b) have four reaction components, whereas we have only 3 equations of equilibrium. Hence the beams in Figs. 1.8(a) and (b) are externally indeterminate to the first degree. Similarly, the beam and frame shown in Figs. 1.8(c) and (d) are externally indeterminate to the 3^{rd} degree.

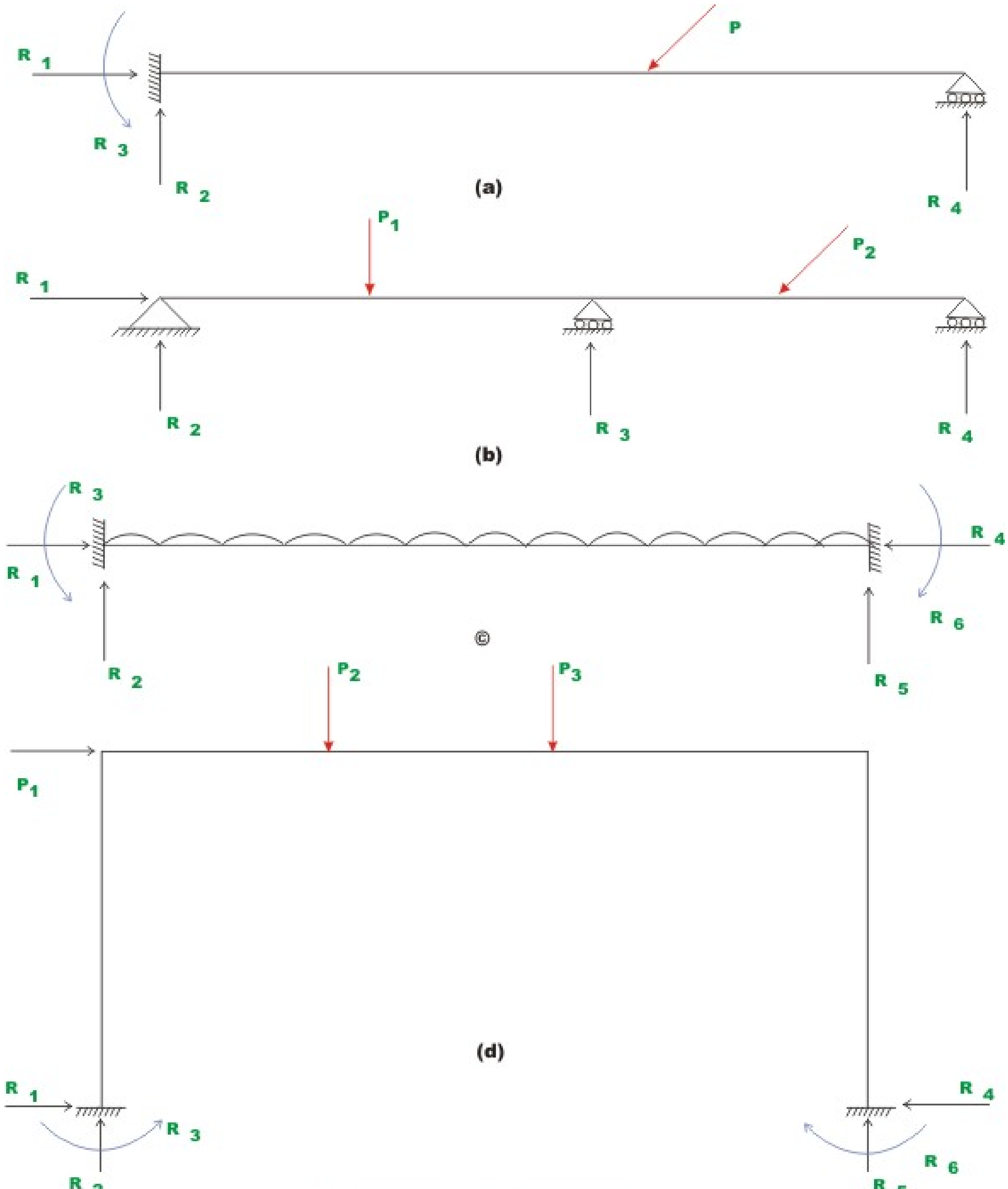
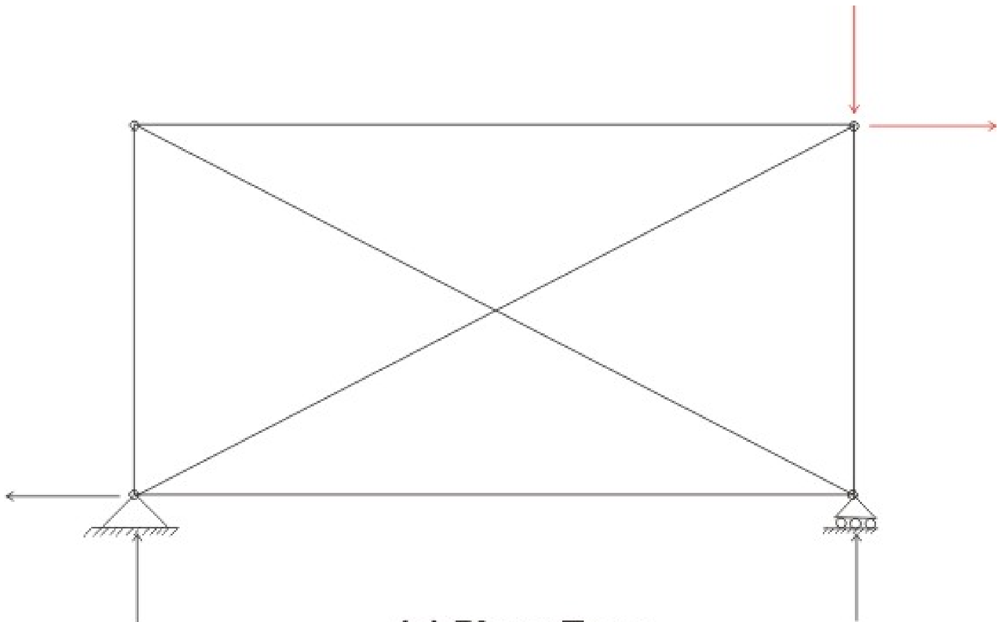
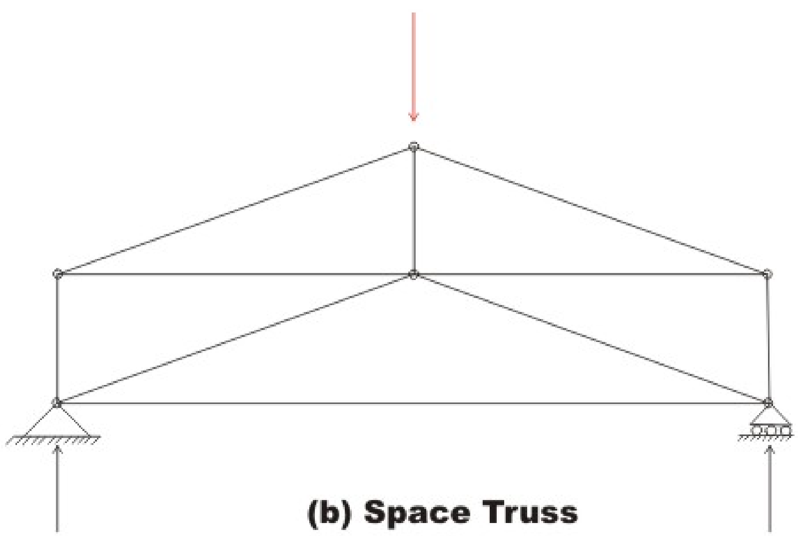


Fig 1.8 Externally Statically Indeterminate Structures

Now, consider trusses shown in Figs. 1.9(a) and (b). In these structures, reactions could be evaluated based on the equations of equilibrium. However, member forces can not be determined based on statics alone. In Fig. 1.9(a), if one of the diagonal members is removed (cut) from the structure then the forces in the members can be calculated based on equations of equilibrium. Thus, structures shown in Figs. 1.9(a) and (b) are internally indeterminate to first degree.The truss and frame shown in Fig. 1.10(a) and (b) are both externally and internally indeterminate.

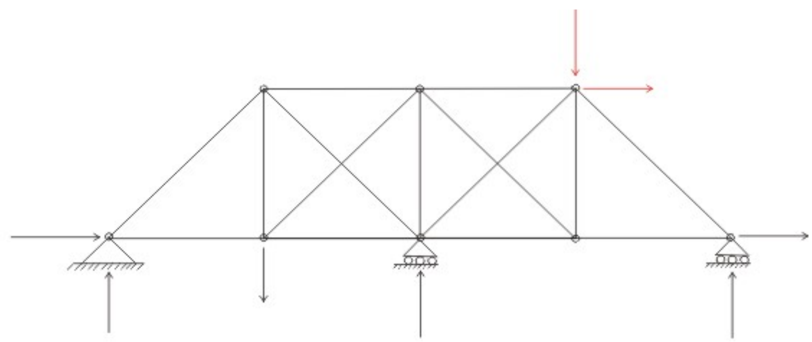


(a) Plane Truss

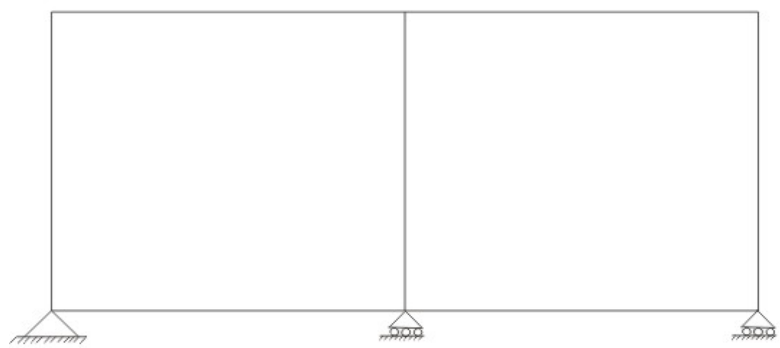


(b) Space Truss

Fig 1.9 Internally Statically Indeterminate Structures



(a) Plane Truss



(b) Plane Frame

Fig 1.10 Externally and Internally Indeterminate Structures

So far, we have determined the degree of indeterminacy by inspection. Such an approach runs into difficulty when the number of members in a structure increases. Hence, let us derive an algebraic expression for calculating degree of static indeterminacy.

Consider a planar stable truss structure having m members and j joints. Let the number of unknown reaction components in the structure be r. Now, the total number of unknowns in the structure is $m+r$. At each joint we could write two equilibrium equations for planar truss structure, viz., $\sum F_x = 0$ and $\sum F_y = 0$. Hence total number of equations that could be written is $2j$.

If $2j = m + r$ then the structure is statically determinate as the number of unknowns are equal to the number of equations available to calculate them.

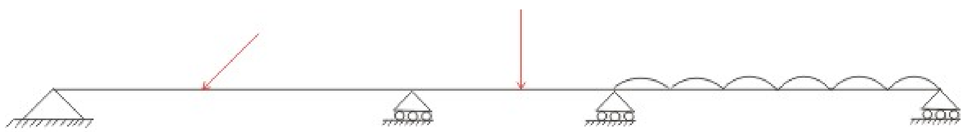
The degree of indeterminacy may be calculated as

$$i = (m+r) - 2j \tag{1.4}$$

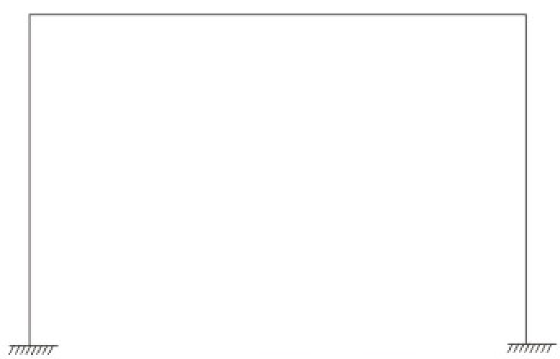
We could write similar expressions for space truss, plane frame, space frame and grillage. For example, the plane frame shown in Fig.1.11 (c) has 15 members, 12 joints and 9 reaction components. Hence, the degree of indeterminacy of the structure is

$$i = (15 \times 3 + 9) - 12 \times 3 = 18$$

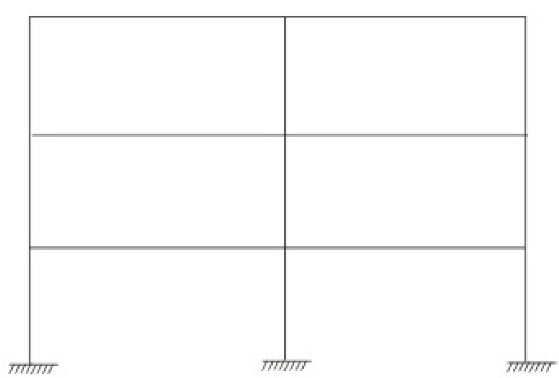
Please note that here, at each joint we could write 3 equations of equilibrium for plane frame.



(a) Continuous Beam



(b) Plane Frame

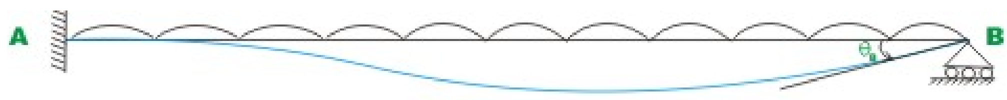


(c) Plane Frame

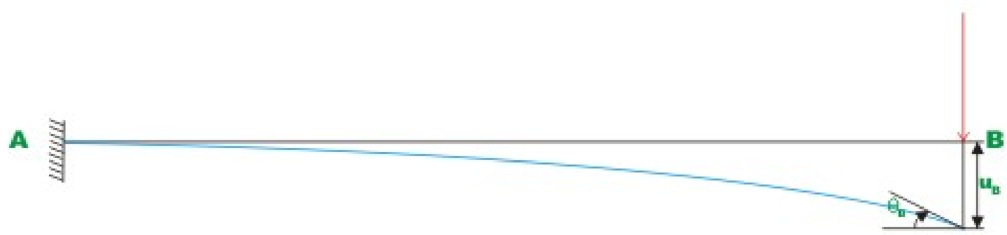
Fig 1.11 Indeterminate Structures

1.5 Kinematic Indeterminacy

When the structure is loaded, the joints undergo displacements in the form of translations and rotations. In the displacement based analysis, these joint displacements are treated as unknown quantities. Consider a propped cantilever beam shown in Fig. 1.12 (a). Usually, the axial rigidity of the beam is so high that the change in its length along axial direction may be neglected. The displacements at a fixed support are zero. Hence, for a propped cantilever beam we have to evaluate only rotation at B and this is known as the kinematic indeterminacy of the structure. A fixed fixed beam is kinematically determinate but statically indeterminate to 3rd degree. A simply supported beam and a cantilever beam are kinematically indeterminate to 2nd degree.



(a) Propped Cantilever Beam



(b) Cantilever Beam

(c) Simply Supported Beam

Fig 1.12 Kinematically Indeterminate Structures

The joint displacements in a structure is treated as independent if each displacement (translation and rotation) can be varied arbitrarily and independently of all other displacements. The number of independent joint displacement in a structure is known as the degree of kinematic indeterminacy or the number of degrees of freedom. In the plane frame shown in Fig. 1.13, the joints B and C have 3 degrees of freedom as shown in the figure. However if axial deformations of the members are neglected then $u_1 = u_4$ and u_2 and u_4 can be neglected. Hence, we have 3 independent joint displacement as shown in Fig. 1.13 i.e. rotations at B and C and one translation.

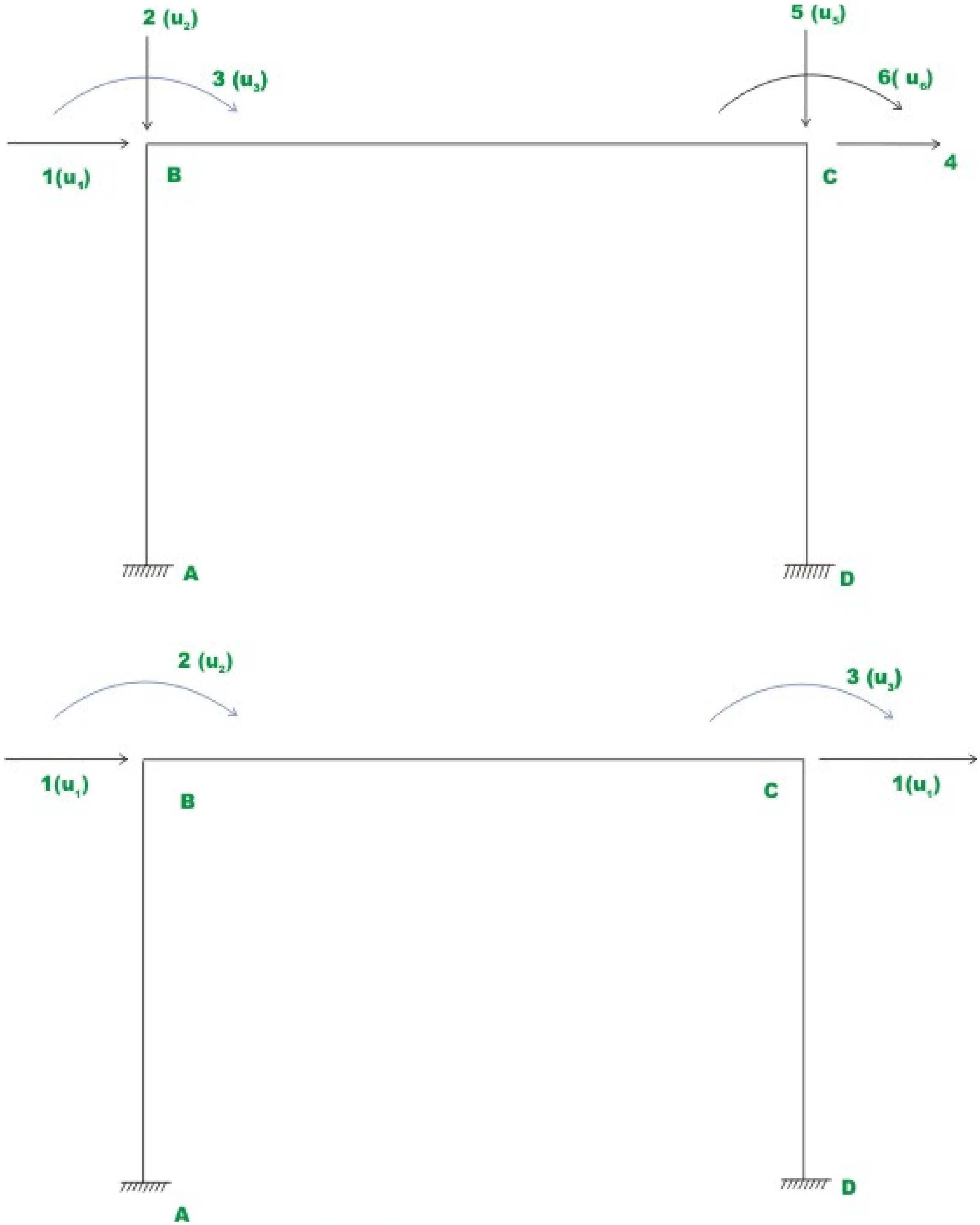


Fig 1.13 Rigid Frame

1.6 Kinematically Unstable Structure

A beam which is supported on roller on both ends (vide. Fig. 1.14) on a horizontal surface can be in the state of static equilibrium only if the resultant of the system of applied loads is a vertical force or a couple. Although this beam is stable under special loading conditions, is unstable under a general type of loading conditions. When a system of forces whose resultant has a component in the horizontal direction is applied on this beam, the structure moves as a rigid body. Such structures are known as kinematically unstable structure. One should avoid such support conditions.

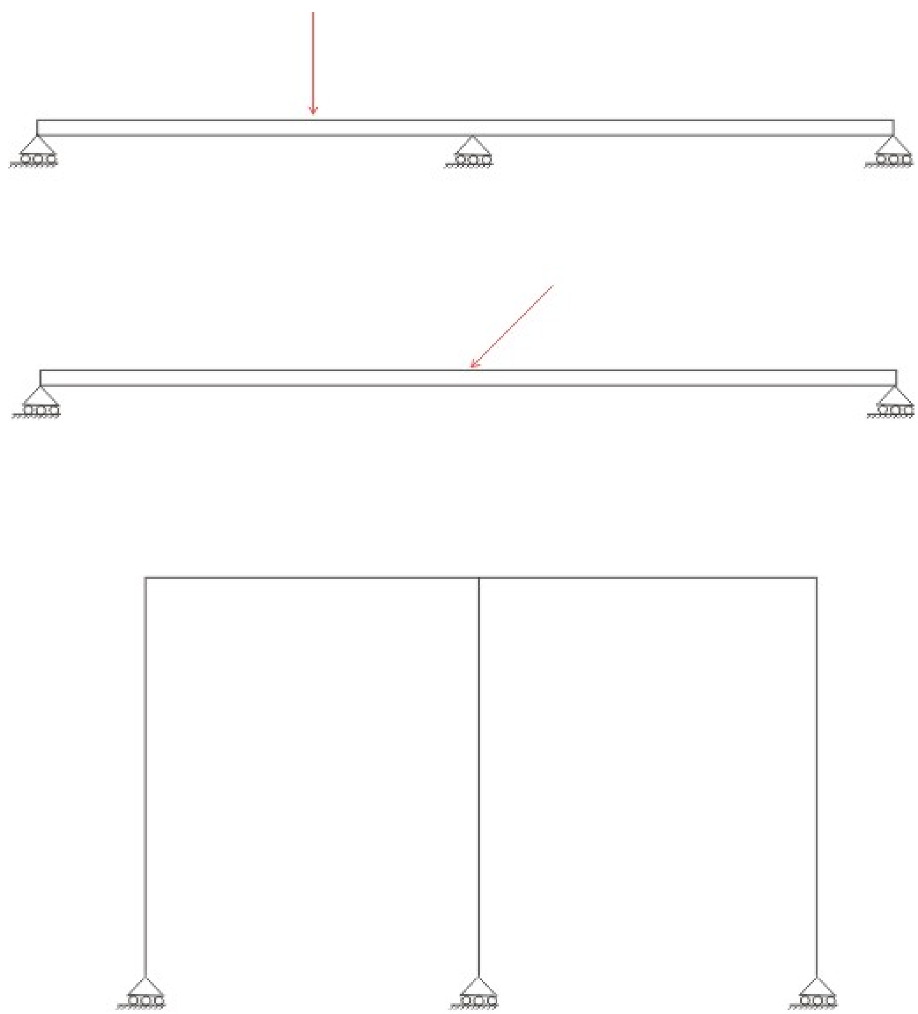


Fig 1.14 Kinematically Unstable Structures

1.7 Compatibility Equations

A structure apart from satisfying equilibrium conditions should also satisfy all the compatibility conditions. These conditions require that the displacements and rotations be continuous throughout the structure and compatible with the nature supports conditions. For example, at a fixed support this requires that displacement and slope should be zero.

1.8 Force-Displacement Relationship

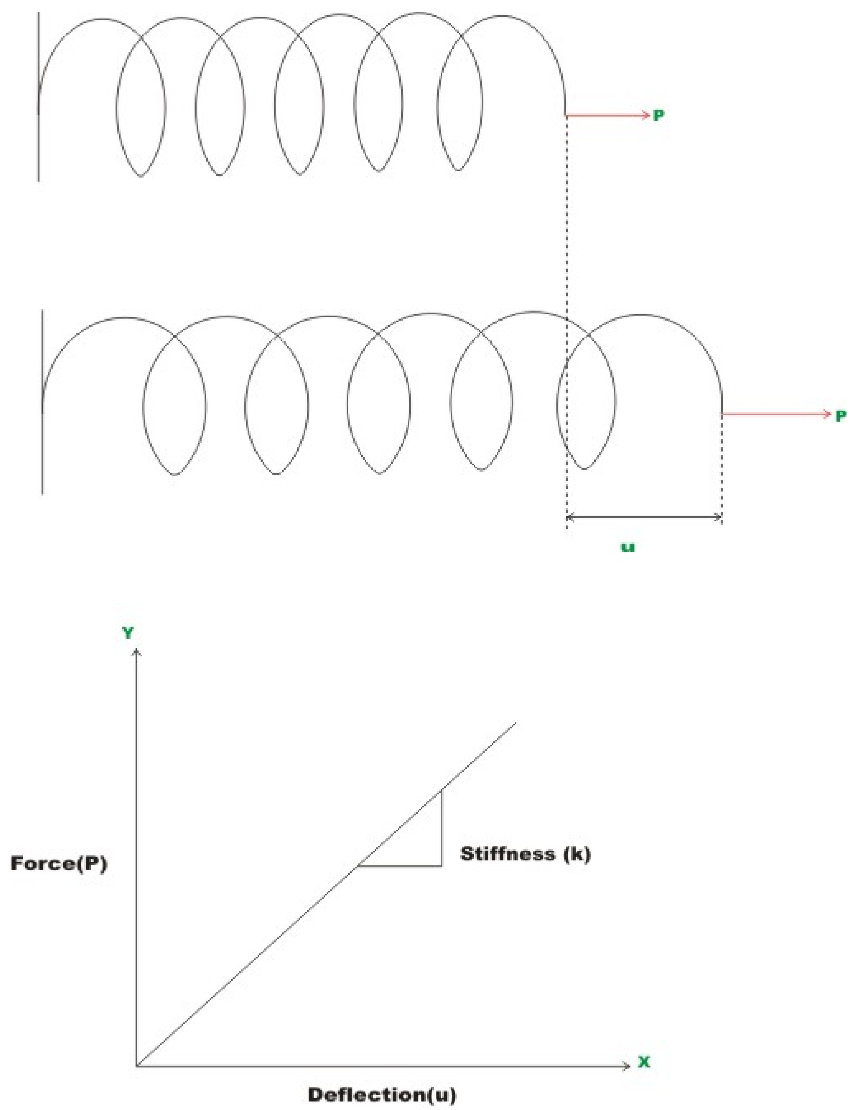


Fig 1.15 Force displacement Relationship

Consider linear elastic spring as shown in Fig.1.15. Let us do a simple experiment. Apply a force P_1 at the end of spring and measure the deformation u_1. Now increase the load to P_2 and measure the deformation u_2. Likewise repeat the experiment for different values of load $P_1, P_2, ..., P_n$. Result may be represented in the form of a graph as shown in the above figure where load is shown on y-axis and deformation on abscissa. The slope of this graph is known as the stiffness of the spring and is represented by k and is given by

$$k = \frac{P_2 - P_1}{u_2 - u_1} = \frac{P}{u} \tag{1.5}$$

$$P = ku \tag{1.6}$$

The spring stiffness may be defined as the force required for the unit deformation of the spring. The stiffness has a unit of force per unit elongation. The inverse of the stiffness is known as flexibility. It is usually denoted by a and it has a unit of displacement per unit force.

$$a = \frac{1}{k} \tag{1.7}$$

the equation (1.6) may be written as

$$P = ku \Rightarrow \qquad u = \frac{1}{k}P = aP \tag{1.8}$$

The above relations discussed for linearly elastic spring will hold good for linearly elastic structures. As an example consider a simply supported beam subjected to a unit concentrated load at the centre. Now the deflection at the centre is given by

$$u = \frac{PL^3}{48EI} \quad \text{or} \quad P = \left(\frac{48EI}{L^3}\right)u \tag{1.9}$$

The stiffness of a structure is defined as the force required for the unit deformation of the structure. Hence, the value of stiffness for the beam is equal to

$$k = \frac{48EI}{L^3}$$

As a second example, consider a cantilever beam subjected to a concentrated load (P) at its tip. Under the action of load, the beam deflects and from first principles the deflection below the load (u) may be calculated as,

$$u = \frac{PL^3}{3EI_{zz}} \tag{1.10}$$

For a given beam of constant cross section, length L, Young's modulus E, and moment of inertia I_{ZZ} the deflection is directly proportional to the applied load. The equation (1.10) may be written as

$$u = a\ P \tag{1.11}$$

Where a is the flexibility coefficient and is $a = \frac{L^3}{3EI_{zz}}$. Usually it is denoted by a_{ij} the flexibility coefficient at i due to unit force applied at j. Hence, the stiffness of the beam is

$$k_{11} = \frac{1}{a_{11}} = \frac{3EI}{L^3} \tag{1.12}$$

1. Internal redundancy

2. External redundancy

In pin jointed plane frames redundancy caused by two many members is called "Internal redundancy".

External redundancy caused by two many supports present in the structure.

Types of Analysis

Analysis

Stiffiness method	Flexibility method
→ Moment disibution method	→ Energy method
→ slope deflection method	→
→	→ column analogy method
→ kanis method	

Differnce b/w determinate and Indeterminate structures

Determinate	Indeterminate
→ Determinate structures can be analysed by using statics alone.	→ Indeterimmnate structures can not be analysed by using statics alone compactibility
→ calculations are very easy	→ calculations are some difficalt.

→ temperature changes donot effect the structure

→ lock of fit cannot produce any stresses

→ If ~~any~~ one member is removed the structure may collapse

→ Temperature changes effect structure.

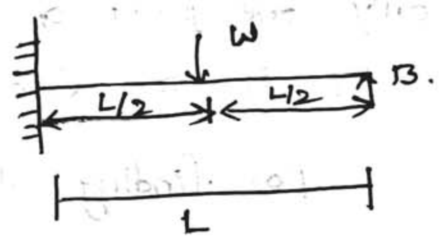
→ Look of fit can produce stresses.

→ If one member is removed the structure may not collapse.

Analysis of proped cantilevers beam by moment area method

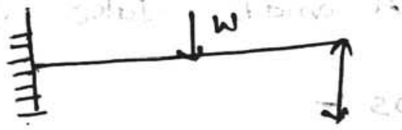
1. Draw B.M diagram for the propped cantilevers loaded as shown in fig. The supports A and B remain at the same level the load.

$$V_A = V_B = \frac{W}{2}$$



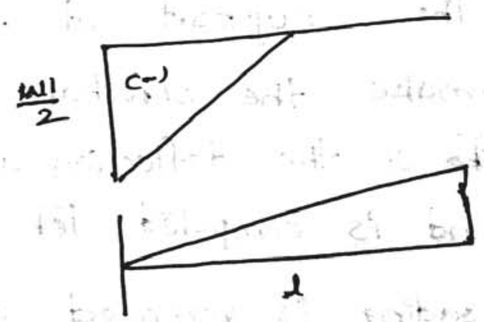
$$M_A = \frac{Wl}{2}$$

$$\delta B_1 = \frac{M_A \times \frac{l}{4} \times \frac{3}{6} l}{EI}$$



$$= \frac{wl}{2} \times \frac{l}{4} \times \frac{5}{6} l \times \frac{1}{EI}$$

$$= \frac{5wl^3}{48EI}$$



$$\delta B_2 = \frac{-R_B l \times \frac{l}{2} \times \frac{2}{3} l}{EI}$$

$$= \frac{-R_B l^3}{3EI}$$

$$\delta B_1 + \delta B_2 = 0$$

$$\frac{5Wl^3}{48EI} - \frac{R_B l^2}{3EI}$$

Moment area method:-

Moment area method is very useful and simple method for finding slopes and deflections of the beam. The method analyise the preopelties of the area x the B.M diagram and also the moment of that area. this method expeatly suiled the deflection and angle of rotation an only one point of the beam is requried

It is a Geometrical method.

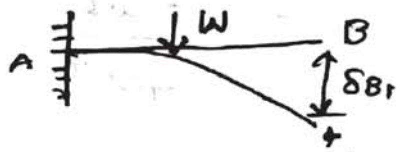
For finding slope $\frac{M}{EI}$ diagram from point A to B

for finding deflection moment of $\frac{M}{EI}$ diagram b/w A and B take about B and centroid of the section

steps :-

→ The support at free end is removed to make the structure determine

→ ~~The~~ it the deflection at the free end is computed let it be δ_1



→ Loading is removed and the reaction at free end is kept as ~~show~~ unknown and deflection at free end is calcuated

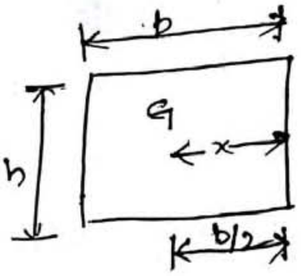
$\delta B_1 + \delta B_2 = 0$

In case support at free end remains as same level when the beam is loaded.

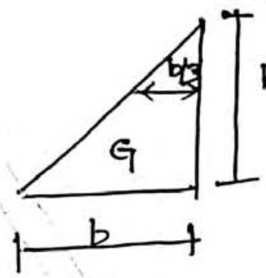
$\delta B_1 + \delta B_2 = 0$

If it is not at same level

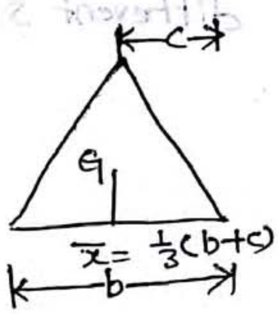
$\delta B_1 + \delta B = \delta$



$A = bh$

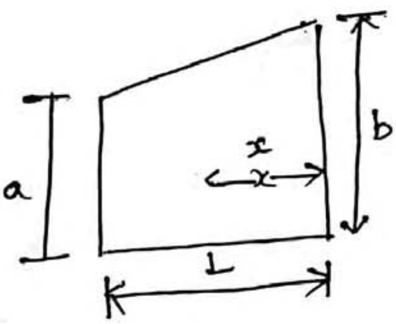


$A = \frac{1}{2} bh$



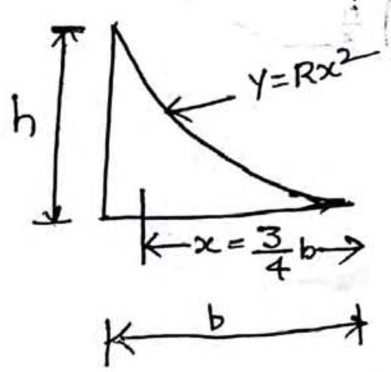
$\bar{x} = \frac{1}{3}(b+c)$

$A = \frac{1}{2} bh$



$\bar{x} = \frac{L}{3}\left[\frac{2a+b}{a+b}\right]$

$A = \frac{L(a+b)}{2}$

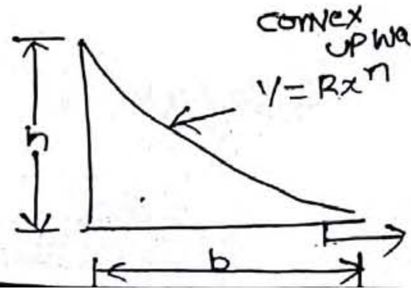


parabolic spandrel

$A = \frac{1}{3} bh$

$x = \frac{3}{4} b$

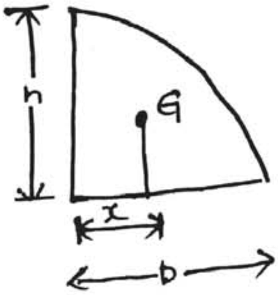
convex upwards → spandrel of n^{th} deg.



$A = \frac{bh}{n+1}$

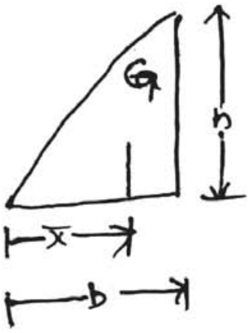
$x = \frac{2}{3} b.$

$\bar{x} = b\left(\frac{n+1}{n+2}\right)$



$$\bar{x} = \frac{b}{2}\left(\frac{n+1}{n+2}\right)$$

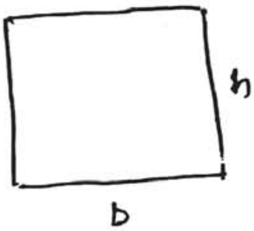
$$A = bh\left(\frac{n}{n+1}\right)$$



$$\bar{x} = \frac{2}{\pi} b$$ sine curve

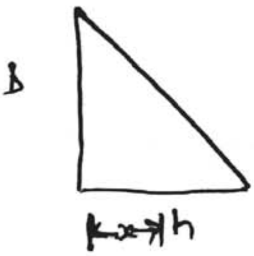
$$A = \frac{2}{\pi} bh$$

Area and centroidal distances for different shapes



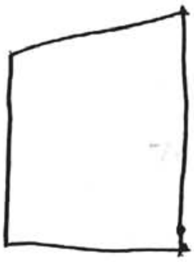
$$A = b \times h$$

$$\bar{x} = \frac{h}{2}$$



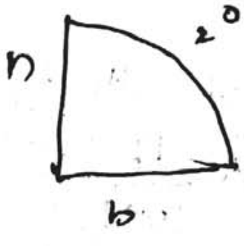
$$A = \frac{1}{2} bh$$

$$\bar{x} = \frac{h}{3}.$$



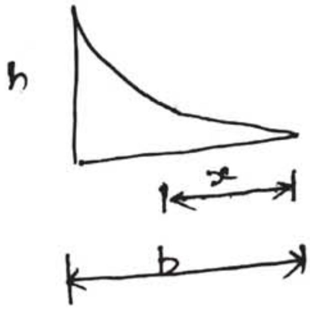
$$A = \left(\frac{a+b}{2}\right) h$$

$$\bar{x} = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$$



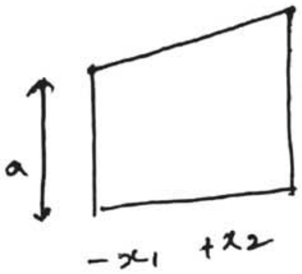
$$A = \frac{2}{3} bh$$

$$\bar{x} = \frac{3}{8} b.$$



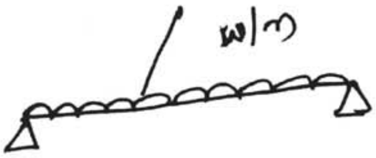
$$A = \frac{1}{3} bh$$

$$\bar{x} = \frac{3}{4} b$$

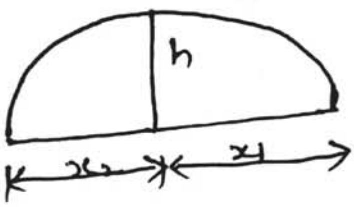


$$x_1 = \left(\frac{2a+b}{a+b}\right) \frac{h}{3}$$

$$x_2 = \left(\frac{a+2b}{a+b}\right) \frac{h}{3}.$$

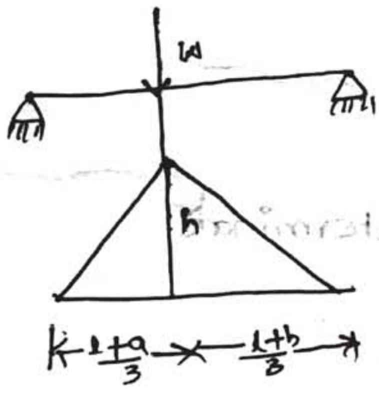


$$A = \frac{2}{3} bh$$



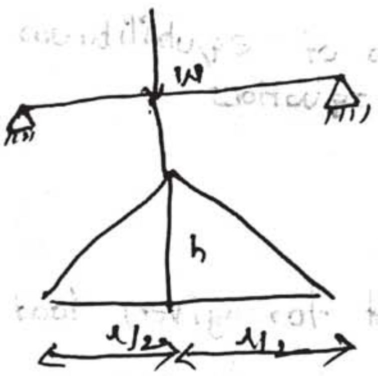
$$x_1 = \frac{5l}{8} \qquad h = \frac{wl^2}{8}$$

$$x_2 = \frac{3l}{8}$$



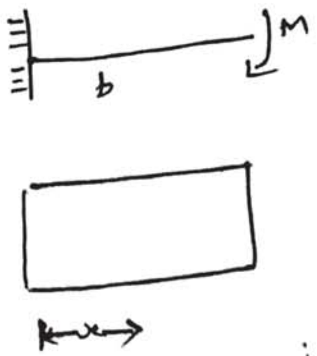
$$A = \frac{1}{2} bh$$

$$h = \frac{wab}{l}$$



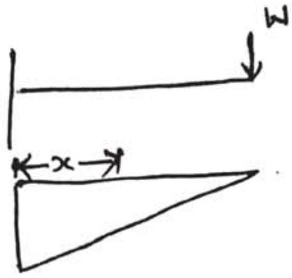
$$A = \frac{1}{2} bh$$

$$h = \frac{wl}{4}$$



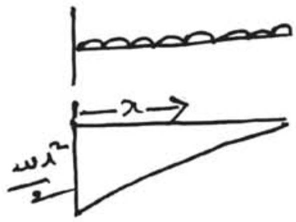
$A = bh$

$x = \frac{1}{2} b$



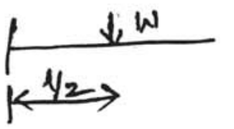
$A = \frac{1}{2} bh$

$x = 2/3\, b.$



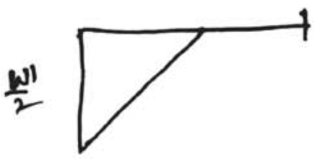
$A = \frac{1}{3} bh$

$x = \frac{1}{4} b$



$A = \frac{1}{2} bh$

$x = \frac{5}{6} b$



$\frac{l}{2} + \left(\frac{2}{3} \times \frac{l}{2}\right) \Rightarrow \frac{5l}{6}.$

Moment area method for indeterminate structure :-

→ Degree of randuancy (DOR)

DOR = No of unknowns - No of equilibrium equations

→ μ diagram

We are finding moment for given load

For the given load we are finding

Area of B.M diagram is called μ diagram

μ diagram denoted by $= A\mu$

→ μ' diagram

For the given reaction not considering load we determine B.M diagram for the reaction is called μ' diagram

Area of μ diagram denoted by $(A\mu')$

→ $A\mu\, x\mu$

→ $A\mu'\, x\mu'$

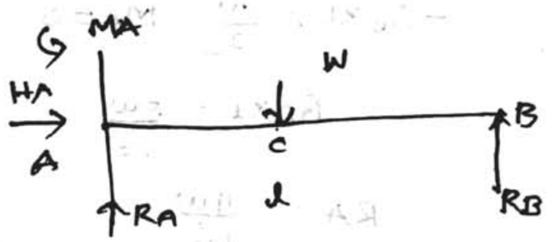
→ $A\mu\, x\mu = A\mu'\, x\mu'$

→ S.F.D

→ B.M.D

→ Deflection.

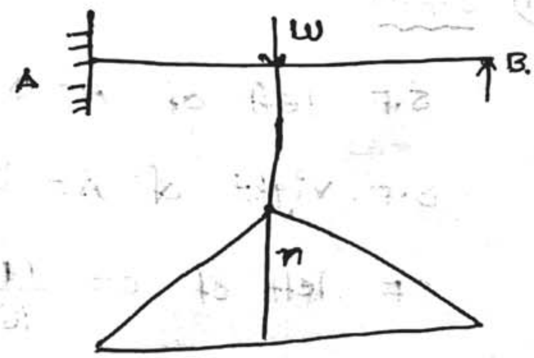
1. Draw B.M.D for the propped cantilever loaded as shown in fig the supports A and B remain at the same level after the load



→ No of unknowns = 4

No of equations = 3

DOR = 1

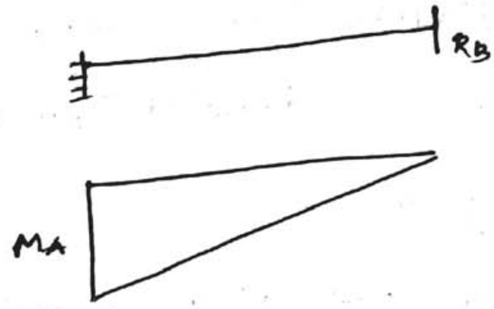


→ u diagram

$h = \frac{Wl}{4}$

$A = \frac{1}{2} \times l \times \frac{Wl}{4}$

3) μ diagram



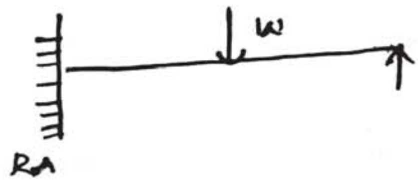
$$A\mu' = \frac{1}{2} bh$$

$$= \frac{1}{2} \times l \times MA$$

4) $A\mu x\mu = \frac{Wl^2}{8} \times \frac{l}{2}$

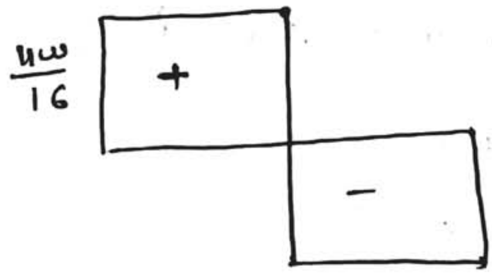
5) $A\mu' x\mu' = \frac{MAl}{2} \times \frac{2}{3} l$

6) $A\mu x\mu = A\mu' x\mu'$



$$\frac{Wl^2}{8} \times \frac{l}{2} = \frac{MAl}{2} \times \frac{2}{3} \times l$$

$$M_A = \frac{3Wl}{16}.$$



S.F.D

$$\Sigma V = 0$$

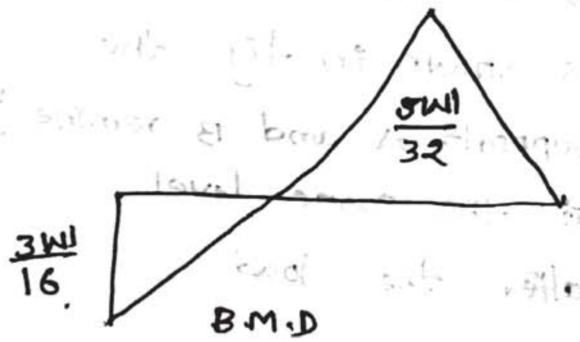
$$RA + RB = \omega$$

$$\Sigma MA = 0.$$

$$-RB \times l + \frac{Wl}{2} - MA = 0$$

$$RB \times l = \frac{5w}{16}.$$

$$RA = \frac{11w}{16}$$



B.M.D

7) S.F.D

S.F left of $A = 0$

S.F right of $A = \frac{11w}{16}$

S.F left of $C = \frac{1 6w}{16}$

S.F right of $C = \frac{11w}{16} - w = \frac{-5w}{16}$

S.F left of $B = -\frac{5w}{16}$

S.F right of $B = 0.$

→ B.M.D.

B.M at A = $M_A = -\frac{3}{16}wl$

B.M at C = $R_A \times \frac{l}{2} - M_A$

B.M at B = $R_A \times l - M_A - \frac{Wl}{2}$

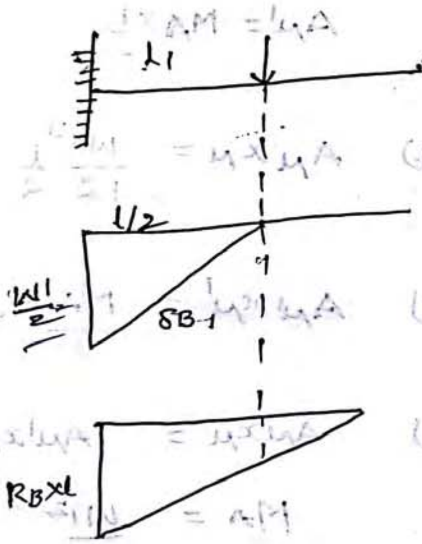
$= 0.$

→ Deflection

$\delta_{B1} + \delta_{B2} = 0$

$\delta_{B1} = A\bar{x} \cdot \frac{1}{EI} = \frac{\frac{1}{2} \times \frac{l}{2} \times \frac{wl}{2} \times \frac{5}{6} l}{EI}$

$= \frac{5wl^3}{48EI}$



$\delta_{B2} = A\bar{x} \cdot \frac{1}{EI} = \frac{-R_B \times l \times \frac{1}{2} l \times \frac{2}{3} l}{EI}$

$= R_B l^3 / 8EI$

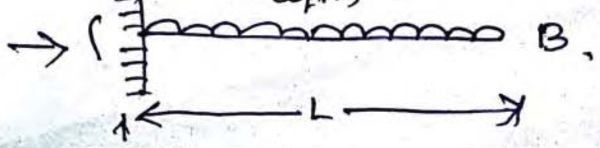
$\frac{5Wl^3}{48EI} - \frac{R_B \times l^3}{3EI} = 0$

$\frac{R_B l^3}{3EI} = \frac{5wl^3}{48EI}$

$R_B = \frac{5w}{16}$

Max BM = $\frac{5w}{16} \times \frac{l}{2} = \frac{5wl}{32}$

2) Draw B.M for the proped cantilever, and S.F diagram loaded as shown in fg the supports A and B drawn at the same level after the load

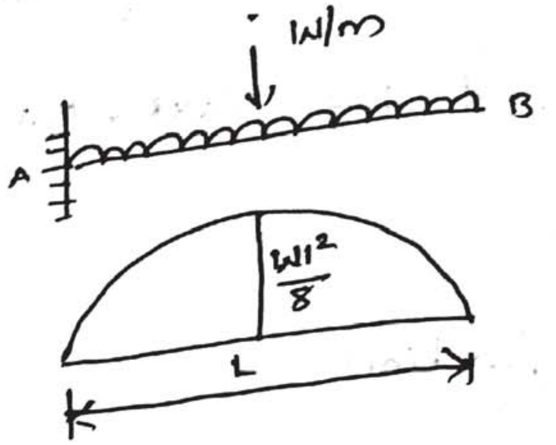


Sol

1 DOR

$= 3-2 = 1$

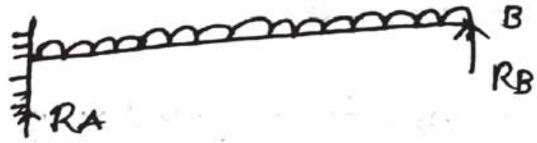
2) M diagram



$h = \frac{wl^2}{8}$

$A_M = \frac{Wl^3}{12}$ $\boxed{\frac{2}{3} \times bh}$

3) M' diagram



$A_{M'} = M_A \times \frac{l}{2}$

4) $A_M x_M = \frac{Wl^3}{12} \frac{l}{2} = \frac{Wl^4}{24}$

5) $A_{M'} x_{M'} = \frac{M_A \cdot l}{2} \times \frac{2}{3} \times l$

6) $A_M x_M = A_{M'} x_{M'}$

$M_A = \frac{Wl^2}{8}$

7) Reactions :-

$R_A + R_B = Wl$

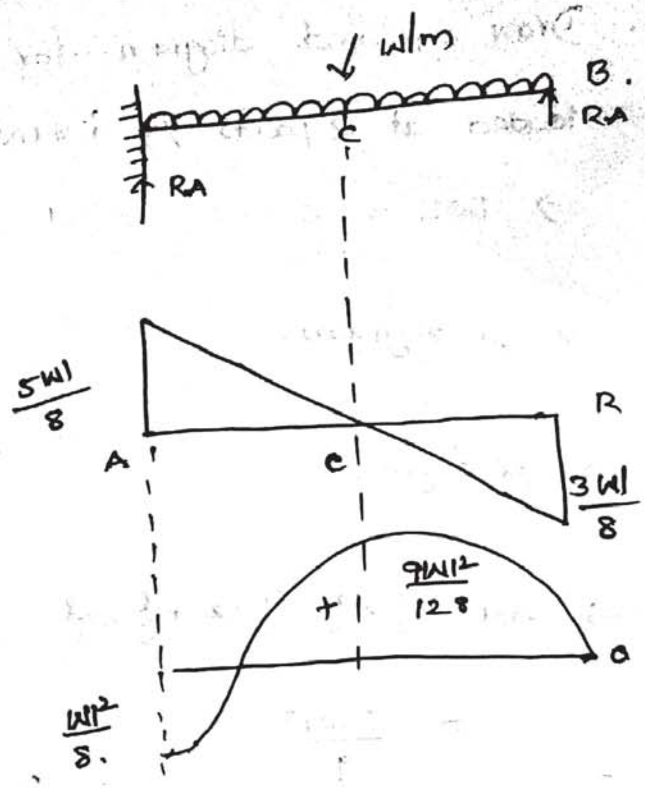
$\Sigma M_A = 0$

$-R_B \times L + \frac{Wl^2}{2} - M_A = 0$

$R_B = \frac{3Wl}{8}$

$R_A = \frac{5Wl}{8}$

8. S.F.D.



$(R_A)L = 0$

$(R_A)R = \frac{5Wl}{8}$

$(R_B)L = -\frac{3Wl}{8}$

$(R_B)R = 0$

9. $\Sigma V = 0$

$R_A = wx$

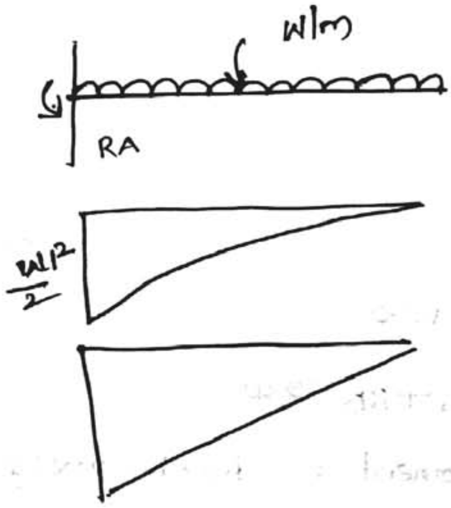
$\frac{5Wl}{8} - wx = 0$

$x = \frac{5l}{8}.$

B.M calcualtions

B.M at A = $-M_A$

B.m at C = $-M_A + R_A - \frac{wx^2}{2}$



Bm at B = 0.

Deflections

$\delta B_1 + \delta B_2 = 0$

$\delta B_1 = A\bar{x}$

$= \frac{Wl^3}{6} * \frac{3}{4} l \cdot \frac{1}{EI}$

$= \frac{Wl^4}{8EI}$

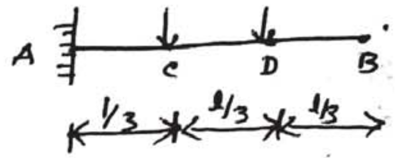
$\delta B_2 = \frac{-R_B \times l^3}{3} = 0$

$R_B = \frac{3Wl}{8}$

Max B.m = $\frac{9wl^2}{128}.$

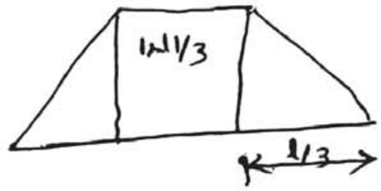
3. Draw S.F and diagram for the propped beam loaded at 2 points $l/3$ distances equally

→ DOR = $5-3=2$, $3-2=1$



→ μ diagram

→ μ' diagrams



→ $A\mu = \frac{1}{2} \times \frac{l}{3} \times \frac{Wl}{3} \times 2 + \frac{l}{3} \times \frac{wl}{3}$

$= \frac{2wl^2}{9}$

→ $A'\mu' = \frac{1}{2} \times M_A l = \frac{M_A l}{2}$

→ $\Sigma V = 0$

$R_A + R_B = 2w$

Moment $= -R_B \times l + w \times \frac{2l}{3} + \frac{wl}{3} - M_A = 0$

$R_B = \frac{2w}{3}$

$R_A = \frac{4W}{3}$

→ S.F calculation

$(V_A)_L = 0$

$(V_A)_R = R_A = \frac{4w}{3}$

$(V_C)_L = \frac{4w}{3}$

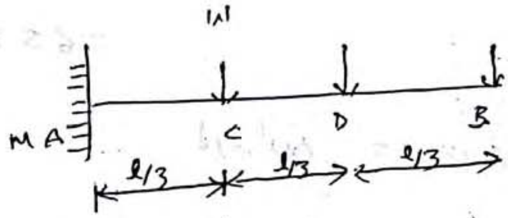
$(V_C)_R = \frac{4w}{w} - w = \frac{w}{3}$

$(V/D)\ L = \frac{W}{3}$

$(V/D)\ R = \frac{w}{2} - w = -\frac{2w}{2}$

$(V/B)\ L = -\frac{2w}{3}$

$(V/B)\ R = 0$

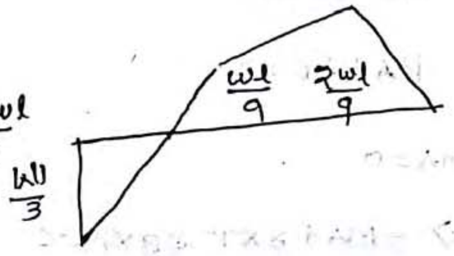


→ B.M calcuation

B.m at A $= -MA = -\frac{Wl}{3}$

B.m at C $= -MA + \frac{4W}{3} \cdot \frac{l}{3} = \frac{wl}{9}$

B.m at O $= -MA + \frac{4w}{3} \cdot \frac{2l}{3} - \frac{wl}{3} = \frac{2wl}{9}$

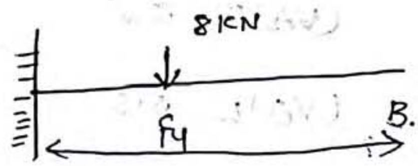


B m at D $= -MA + \frac{4w}{3} \cdot \frac{2l}{3} - \frac{wl}{3}$

$= \frac{2wl}{9}$

B.m at B $= 0$,

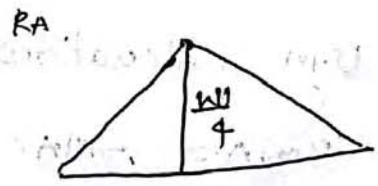
4. Draw S.F and B.M diagram for the beam fixed at A and simply supported at B and a compacled load of 8 KN acting at the mid span the total length remains at the same level after the application of find deflection also.



→ DOR $= 4 - 3 = 1$

→ μ diagram

$h = \frac{wl}{4} = \frac{8 \times 14}{4} = 28$



→ μ' diagram

$A\mu' = \frac{1}{2} bh = \frac{1}{2} \times 14 \times MA = 7MA$

→ $A\mu\, x_\mu = 196 \times \frac{l}{2} = 196 \times \frac{14}{2} = 1372$

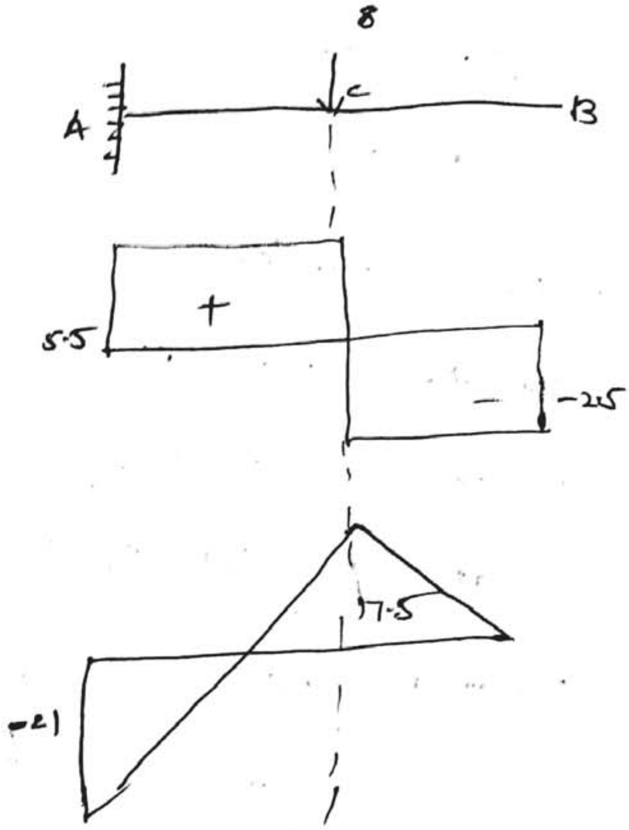
$\rightarrow A_{M'} x_{M'} = 7M_A \times \frac{2}{3} l = 7M_A \times \frac{2}{3} \times 14$

$= 65.3M_A$

$\rightarrow A_M x_M = A_{M'} x_{M'}$

$1372 = 65.3M_A$

$M_A = 21 \, kNm$



$\rightarrow$ Reactions

$R_A + R_B = 8$

$M_A = 0$

$\Rightarrow -M_A + 8 \times 7 - R_B \times 14 = 0$

$\Rightarrow -21 + 56 - 14R_B = 0$

$\Rightarrow 14R_B = 35$

$R_B = 2.5$

$R_A = 5.5$

$\rightarrow$ S.F calcuations

$(V_A)_L = 0$ $\quad$ $(V_B)_L = -2.5$

$(V_A)_R = 5.5$ $\quad$ $(V_B)_R = 0$

$(V_C)_L = 5.5$

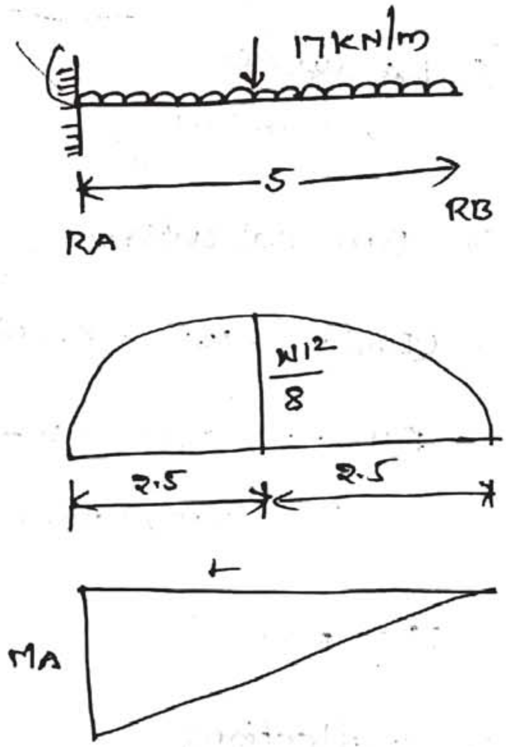
$\rightarrow$ B.M calcuations

$(BM)_A = -M_A = -21$

$(BM)_C = -M_A + R_A \times 7 = 17.5$

$(BM)_B = 0.$

5. A beam AB 5mt long is fixed at A and simply supported at B the beam carries a uniformly distributed at 17KN/m. Find what couple should be applied at B. so that bending monent at A is zero central deflection is = $\frac{Wl^3}{185EI}$



→ DOR = 3-2=1

→ M diagram

$A_M = \frac{2}{3} bh$

$= \frac{2}{3} \times 5 \times 53.125$

$= 177.08 m^2$

→ M' diagram

$A_{M'} = \frac{1}{2} \times MA \times l = 2.5 MA$

$x_{M'} = \frac{2 \times 5}{3} = 3.33$

→ $A_M x_M = A_{M'} x_{M'}$

$442.708 = 53.146 KNm$

→ Reactions

$RA + RB = 17 \times 5$

$RB \times l - MA + \frac{Wl^2}{2} = 0$

$-RB \times 5 - 53.146 + 212.5 = 0$

$-RB \times 5 = -154.35$

$RB = 31.31 KN$

$RA = 53.13 KN.$

→ S.F calcualations

$(V_A)_L = 0$

$(V_A)_R = 53.23\,kN$

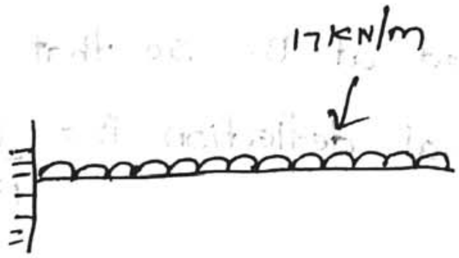
$(V_B)_L = R_A - Wl$

$(V_B)_R = 0$

$V = 0.$

$R_A - wx = 0$

$x = 3.125\,mts$



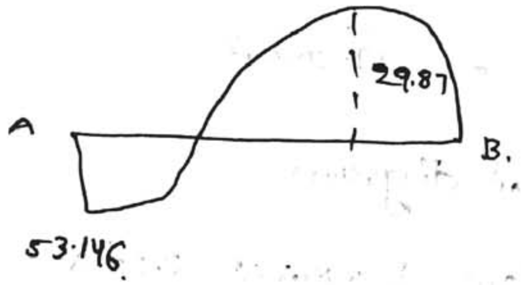
→ Bm calcuations

$(BM)_A = -M_A = -53.46\,KNm$

$(Bm)_C = -M_A + R_A \times x - \frac{wx^2}{2}$

$= 29.87\,kNm$

$(Bm)_B = 0$



→ Deflections

$\delta_{B_1} = \frac{WL^4}{8EI} = \frac{1328.125}{EI}$

$\delta_{B_2} = \frac{R_B x^3}{3EI} = -41.66 R_B$

$\delta_{B_1} + \delta_{B_2} = 0$

$R_B = 31.875\,kN$

UNIT-2

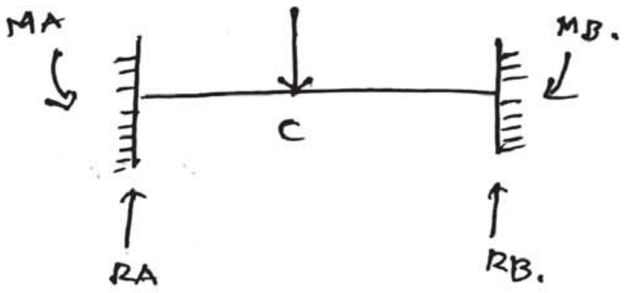
FIXED BEAMS

A beam whose both ends are fixed is called as a fixed beam (or) built in beam or encastered beam.

In the case of a fixed beam, the slope and deflections are zero. But fixed end subject to end moments Hence end moments not equal to zero.

Moment area method:-

→ find the fixed end moments for a fixed beam of span l subjected to a concentrated load w at midspan which is shown in fig.



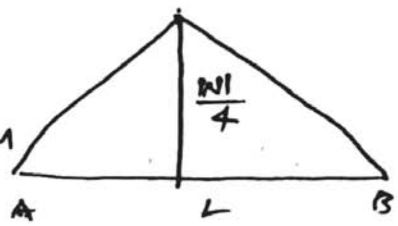
→ DOR = 4-2=2

→ μ diagram

$A\mu = \frac{1}{2} \times l \times \frac{wl}{4}$

→ μ' diagram

$A\mu' = M \times l$ [∵ $M_A = M_B = M$



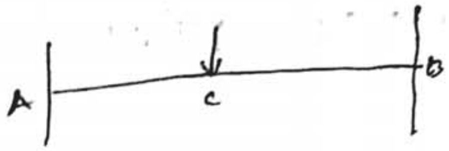
→ $A\mu = A\mu'$

$\frac{Wl^2}{8} = Ml$

$M = \frac{wl}{8}$

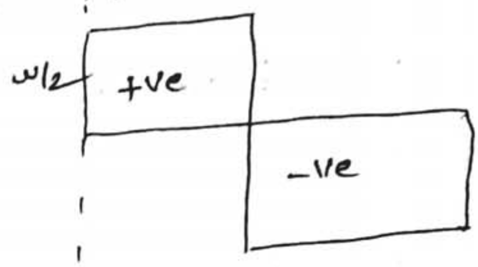
Reactions

$R_A + R_B = \omega$



$R_A = \frac{\omega}{L} = R_B$

SF calcualtions



$(V_A)_L = 0$

$(V_B)_L = -\frac{\omega}{2}$

$(V_B)_R = 0$

$(V_C)_L = \frac{\omega}{2}$

$(V_C)_R = -\frac{\omega}{2}$

B.M calcuations:-

$BM \text{ at } A = -M_A = \frac{-\omega l}{8}$

$B.m \text{ at } c = -M_A + R_A \cdot \frac{l}{2}$

$= \frac{-Wl}{8} + \frac{\omega}{2} \cdot \frac{l}{2}$

$= \frac{\omega l}{8}$

$B.m \text{ at } B = -M_A + R_A \cdot L - \omega \cdot \frac{l}{2}$

$= -\frac{Wl}{8} + \frac{Wl}{2} - \frac{\omega l}{2}$

$= \frac{-\omega l}{8}$

Deflections :-

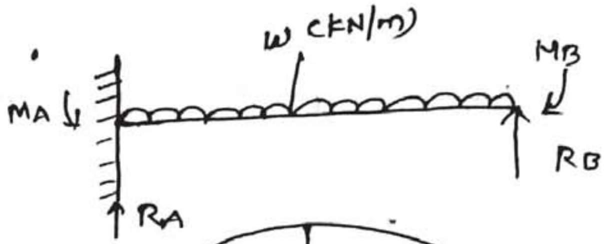
$\varepsilon A\bar{x} = \frac{1}{2} \times \frac{l}{2} \times \frac{Wl}{4} \times \frac{1}{3} \times \frac{l}{2} + M \times \frac{l}{2} \times \frac{l}{4}$

$= \frac{Wl^2}{16} \times \frac{l}{6} + \frac{Ml^2}{8}$

$$= \frac{Wl^3}{96} + \frac{Ml^2}{8} = \frac{wl^3}{96} - \frac{wl^3}{64} = \frac{Wl^3}{192}$$

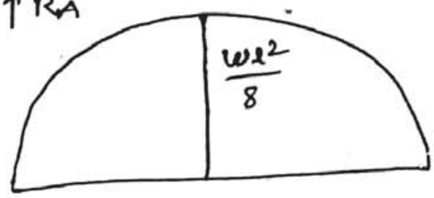
$$\delta_B = \frac{-wl^3}{192EI}$$

(2). Find the fixed end moments for a fixed beam of span l subjected to a uniformly distributed load over the entire span and also find the deflection of beam.



→ DOR = 2.

→ μ diagram



$$A_\mu = \frac{2}{3} \times l \times \frac{wl^2}{8}$$

$$= \frac{wl^3}{12}$$

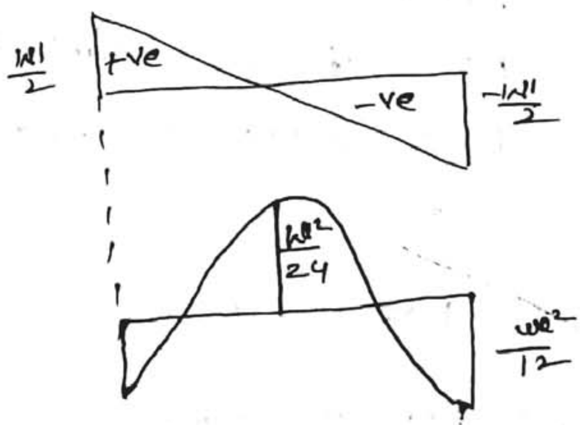
→ μ' diagram

$A_{\mu'} = M \times l$

→ $A_\mu = A_{\mu'}$

$$\frac{wl^3}{12} = M \times l$$

$$M = \frac{wl^2}{12}$$



Reactions

$R_A + R_B = wl$

$R_A = \frac{wl}{2} = R_B$

S.F Calcuation

$(V_A)_L = 0$

$(V_A)_R = \frac{wl}{2}$

$(V_B)_L = \frac{wl}{2} - wl = -\frac{wl}{2}$, $(V_B)_R = 0.$

→ Bm calculations

$$BMA = -MA = \frac{-wl^2}{12}$$

$$BMB = -MA + wl + \frac{l}{2}$$

$$= -\frac{wl^2}{12} + \frac{wl^2}{2}$$

$$= \frac{5wl^2}{12} - \frac{wl^2}{2} = \frac{-wl^2}{12}$$

$$Bmc = RA \times \frac{l}{2} - MA - w\left(\frac{1}{2} \times \frac{l}{4}\right)$$

$$= \frac{wl^2}{4} - \frac{wl^2}{12} - \frac{wl^2}{8} = \frac{wl^2}{24}.$$

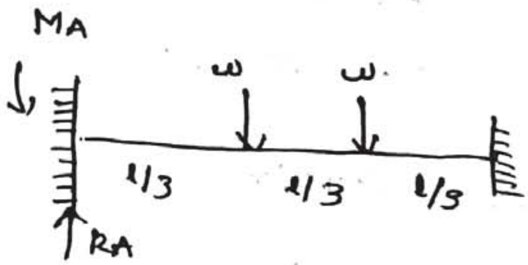
Deflections

$$\Sigma A\bar{x} = \frac{2}{3} \times \frac{l}{2} \times \frac{wl^2}{8} \times \left(\frac{3}{8} \times \frac{l}{2}\right) - M \times \frac{1}{2} \times \frac{l}{4}$$

$$= \frac{wl^4}{128} - \frac{wl^4}{96}$$

$$= \frac{-wl^4}{384}.$$

3. Find the fixed end moments for a fixed beam of span subjected to two concentrated loads located at 1/3rd distances from each end.



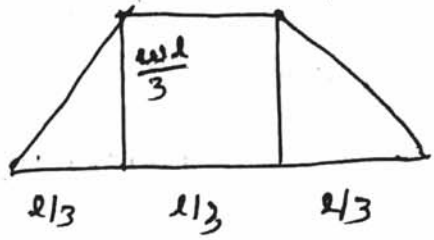
→ DOR = 4-2 = 2

→ μ diagram

$$A\mu = \frac{1}{2}\left(l + \frac{l}{3}\right)\frac{wl}{3}$$

→ μ' diagram

$A_{\mu'} = M.l$

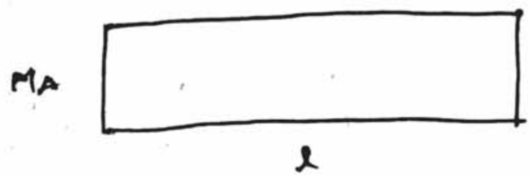


→ $A_M = A_{\mu'}$

$$M.l = \frac{wl^2}{6} + \frac{wl^2}{18}$$

$$= \frac{wl}{6} + \frac{wl}{18}$$

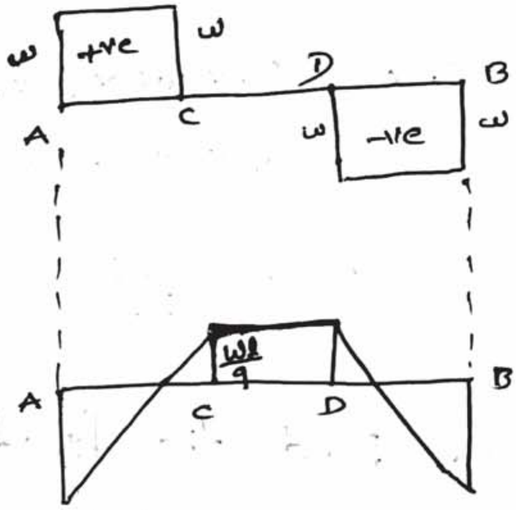
$$= \frac{2wl}{9}.$$



Reactions

$R_A + R_B = 2w$

$R_A = R_B = w$



S.F calcuations

$(V_A)_L = 0$

$(V_A)_R = w$

$(V_B)_L = -w$

$(V_C)_L = w$

$(V_C)_R = 0$

$(V_D)_L = 0$

B.M. calcuations :-

$$(BM)_A = -M_A = -\frac{2wl}{9}$$

$$(BM)_C = -M_A + R_A \times \frac{l}{3} = -\frac{2wl}{9} + \frac{wl}{3} = \frac{wl}{9}$$

$$(BM)_B = -M_A + R_A \times \frac{2l}{3} - w.\frac{l}{3}$$

$$= -\frac{2wl}{9} + \frac{2wl}{3} - \frac{wl}{3} = \frac{wl}{9}$$

$$(BM)_B = -M_A + R_A \times L - w \times \frac{2l}{3} - w \times \frac{l}{3}$$

$$= -\frac{2wl}{9} + wl - wl$$

$$= \frac{-2wl}{9}.$$

Deflection :-

$\Sigma A\bar{x}$

$$= \frac{1}{2}(l+l/3)\frac{wl}{3}\times\frac{l}{2} - M\times\frac{l}{2}\times\frac{l}{4}$$

$$= \left(\frac{wl^2}{6}+\frac{wl^2}{18}\right)\frac{l}{2} - \frac{Ml^2}{8}$$

$$= \frac{wl^3}{12}+\frac{wl^3}{36} - \frac{2wl\times l^2}{9\times 8}$$

$$= \frac{6wl^3}{72}$$

$$\Sigma A\bar{x} = 2/9\times wl\times\frac{l}{2}\times l/4 - \frac{wl}{3}\times\frac{l}{6}\times\frac{l}{12} - \frac{1}{2}\times\frac{wl}{3}\times\frac{2}{3}\left(\frac{l}{6}+\frac{l}{9}\right)$$

$$= \frac{5}{648}\ \frac{wl^3}{EI}$$

4. Find the fixed beam end moments for a fixed beam of span 15 mts. subjected to two concentrated loads 10kN at 1/3rd distances for each end.

→ DOR = 4-2=2

→ μ diagram

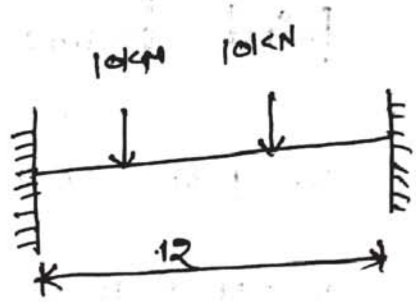
$$A\mu = 1/2(12+4)\times\frac{10\times12}{2}$$

→ μ' diagram

$$A\mu' = M_A\times12$$

$$=12M_A$$

→ $A_M = A_M'$

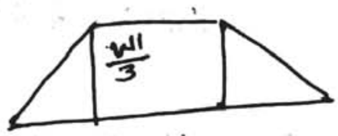


$320 = 12 M_A$

$M = 26.67$

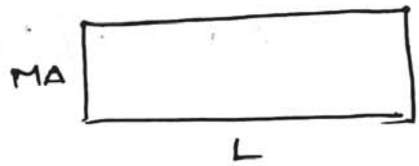
→ Reactions

$R_A + R_B = 20kN$



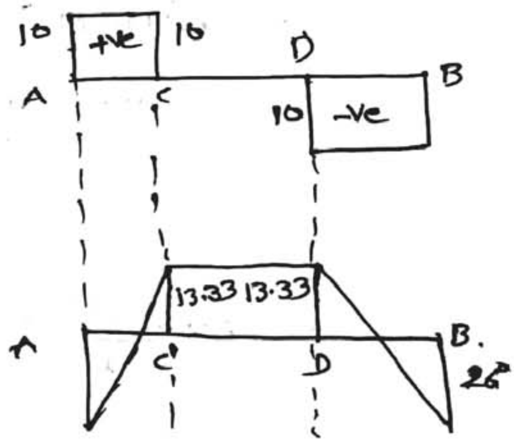
$R_A = 10kN$

$R_B = 10kN$



→ SFD

→ BMD calculations



$(BM)_A = -M = -26.67 kNm$

$(Bm)_C = -M + 10 \times 4$

$= 13.33\ kNm$

$(Bm)_D = 13.33\ kNm$

$(Bm)_B = -26.67\ kNm$

Deflections

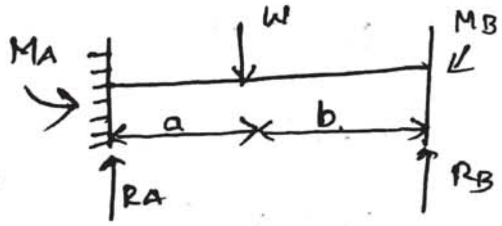
$$EA\bar{x} = \frac{5}{648}\frac{wl^3}{EI}$$

$$= \frac{5}{648} \times \frac{10 \times 12^3}{EI}$$

$$= \frac{133.33}{EI}$$

Find beams with ecentric point loads

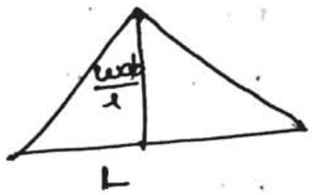
$\rightarrow$ DOR = 4-2=2



$\rightarrow$ M diagram

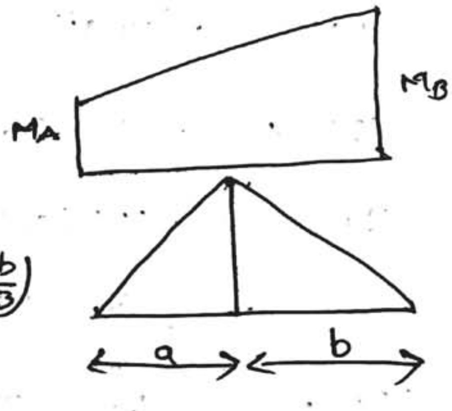
$$A_M = \frac{1}{2} bh = \frac{1}{2} \times \frac{wab}{l} \cdot \frac{wab}{2}$$

$\rightarrow$ M' diagram



$$A_{M'} = -\left(\frac{M_A + M_B \times l}{2}\right)$$

$\rightarrow$ $A_M x_M$



$$= A_{M1} x_{M1} + A_{M2} x_{M2}$$

$$= \frac{1}{2} \times a \times \frac{wab}{l}\left(\frac{2}{3}a + \right.$$

$$\left.\frac{1}{2} b \times \frac{wab}{l}\left(a + \frac{b}{3}\right)\right)$$

$$= \frac{wa^3 b}{3l} + \frac{wa^2b^2}{2l} + \frac{wab^3}{6l}$$

$$= \frac{wab}{l}\left[\frac{a^2}{3} + \frac{ab}{2} + \frac{b^2}{6}\right]$$

$$= \frac{wab}{l}\left[\frac{2a^2 + 3ab + b^2}{6}\right] = \frac{wab}{6}(2a+b)$$

$\rightarrow$ $A_M x_{M'}$

$$= -\left(\frac{M_A + M_B}{2} \times l\right)\left[\frac{2}{3}\left(\frac{2M_A + M_B}{M_A + M_B}\right)\right]$$

$$= \frac{-L^2}{6}\left(2M_A + M_B\right)$$

$\rightarrow$ $A_\mu x_\mu + A_{\mu'} x_{\mu'} = 0 \longrightarrow ①$

$A_\mu + A_{\mu'} = 0 \longrightarrow ②$

$\frac{wab}{6}(2a+b) - \frac{L^2}{6}(2M_A + M_B) = 0 \longrightarrow ③$

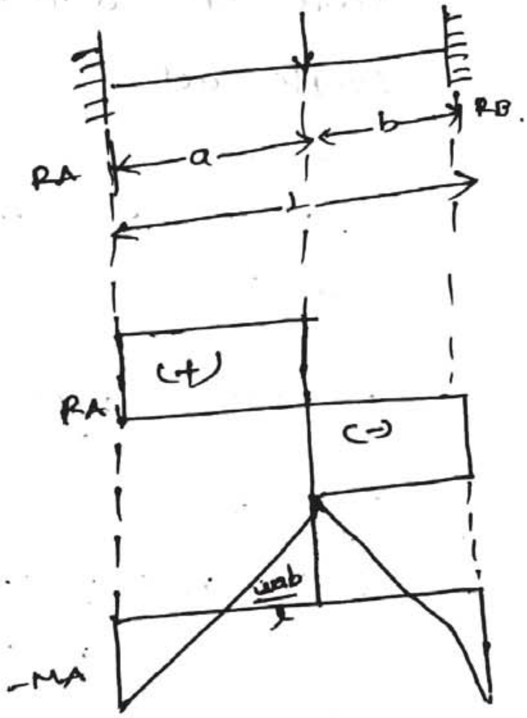
$\frac{wab}{2} - \left(\frac{M_A + M_B}{2}\right)L = 0$

$M_A + M_B = \frac{wab}{L} \longrightarrow ④$

solve ③ & ④

$M_A = \frac{wa^2b}{l^2}$

$M_B = \frac{wab^2}{l^2}$



$\rightarrow$ End reactions

$R_A + R_B = w$

∴ Taking moment about B

$R_A \times l - wb - M_A + M_B = 0$

$R_A = \frac{wb^2}{L^3}(3a+b)$

$R_B = \frac{wa^2}{L^3}(a+3b)$

$\rightarrow$ S.F calcuations

$(V_A)_L = 0$

$(V_A)_R = 0$

$(V_B)_L = R_A - w = -R_B$

$(V_B)_R = 0$

$\rightarrow$ BM calcuations

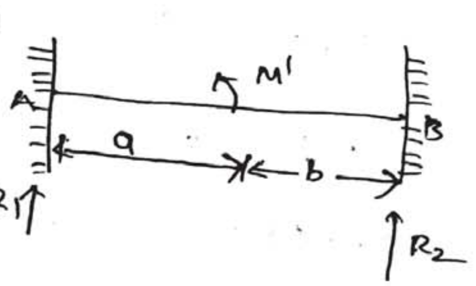
$BM_A = -M_A = -\frac{wab^2}{l^2}$

$$BM_x = \frac{wbx}{l}\left(-M_A + (M_B - M_A)\right) \times \frac{x}{l} = \frac{wab}{l}$$

$$BM_B = \frac{-wba^2}{l^2} L.$$

3. Determine the fixed end moments and reactions for a fixed beam loaded by a couple of m at a distance of 'a' from the left end also find the deflection at point c where the couple acts.

$\Sigma A = 0$



$\frac{1}{2} R_1 L \times L - M_1 L - M'b = 0 \rightarrow (1)$

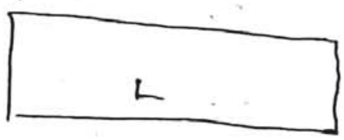
$(1) \times \frac{2}{L^2}$

$$R_1 - \frac{2M_1}{L} = \frac{2M'b}{L^2}$$

$\Sigma A\bar{x} = 0$

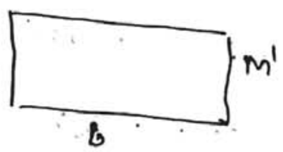
$\frac{1}{2} \times R_1 L \times L (L/3) - (M_1 L) \frac{L}{2}$

$- M'b(b/2) = 0 \rightarrow (3)$



$$R_1 - \frac{3M_1}{L} = \frac{3M'b^2}{L^3} \rightarrow (4)$$

Solve (2) & (4)



$$2R_1 - \frac{5M_1}{L} = \frac{2M'b}{L^2} + \frac{3Mb^2}{L^3}$$

$$= \frac{2M_1 b}{L^2}\left[1 + \frac{3}{L} \right]$$

$$R_1 - \frac{2M_1}{L} = \frac{2M'b}{L^2}$$

$$R_1 - \frac{3M_1}{L} = \frac{3M'b^2}{L^3}$$

$$\frac{M_1}{L} = \frac{2M'b}{L^2} - \frac{3M'b^2}{L^3}$$

$$\rightarrow M_1 = \frac{2M'b}{L} - \frac{3M'b}{L^2} = \frac{M'b}{2}(2a-b)$$

$$\boxed{R_1 = \frac{6Mab}{L^3}} \qquad \boxed{R_2 = -\frac{6M'ab}{L^3}}$$

Taking moment about A

$$-M_1 + M_2 - M' - R_2 L = 0$$

$$M_2 = M_1 + M' + R_2 L$$

$$= \frac{M'b}{L^2}(2a-b) + M' - \frac{6M'ab}{L^3} \times L$$

$$= \frac{M'2ab - M'b + M'L^2 - 6M'ab}{L^2}$$

$$= \frac{-4M'ab + M'(L^2 - b^2)}{L^2}$$

$$= \frac{-4M'ba + M'(2a+b)a}{L^2}$$

$$= \frac{M'a(a-2b)}{L^2}$$

$$y_c = \Sigma A\bar{x} = \frac{1}{2} x a + R.a(a/3) - M_1(a)(a/2)$$

$$= \frac{R_1 a^3}{6} - \frac{M_1 a^2}{2}$$

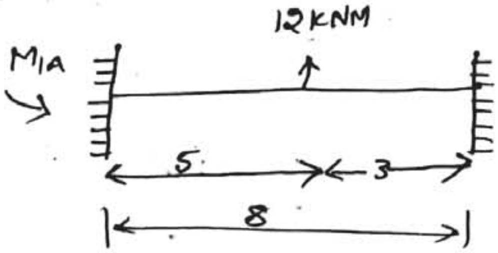
$$= \frac{6M'ab}{L^3 6} \times a^3 - \frac{M_1 a^2}{2}$$

$$= \frac{M'a^4 b}{L^3} - \frac{M'b}{2}(2a-b)a^2$$

$$= \frac{M'a^4 b}{L^3} - \frac{M'a^2 b}{2L^2}(2a-b)$$

$$= \frac{M'a^4 b}{L^3} - \frac{2M'a^3 b + M'a^2 b^2}{2L^2}$$

4. Determine the fixed end moments and reactions for a fixed end beam loaded by a couple of 12KNm at a distance of 5m from the left end the total span of the structure is 8m also find the deflection at the point where the couple acts.



$$M_1 = \frac{M'b}{L^2}(2a-b)$$

$$= \frac{12\times 3}{8^2}(2\times 5-3)$$

$$= 3.93.$$

$$M_2 = \frac{Ma(a-2b)}{L^2} = \frac{12\times 5(5-2\times 3)}{8^2} = -0.9375$$

$$R_1 = \frac{6M'ab}{L^3} = \frac{6\times 12\times 5\times 3}{8^3} = 2.10.$$

$$R_2 = 2.10$$

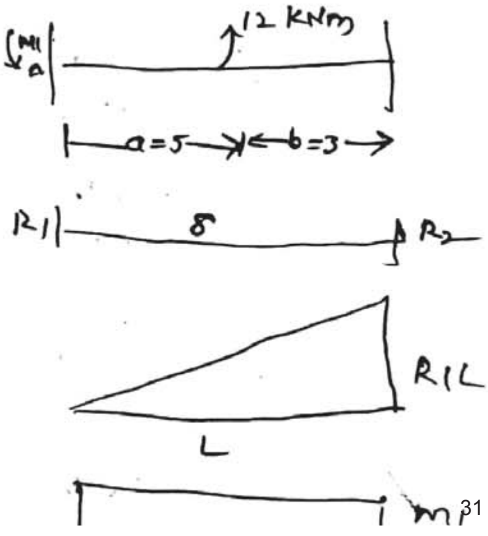
$$y_c = \frac{M'a^2b^2(b-a)}{2EI3} = \frac{12\times 5^2\times 3^2(3-5)}{2\times 8^3} = -5.27$$

$\Sigma A = 0$ summation of areas

$$= \frac{1}{2}\times R_1L\times L - M_1\times L - M'b = 0$$

$$= \frac{1}{2}R_1\times 8\times 8 - M_1\times 8 - 12\times 3 = 0$$

$$= 32R_1 - 8M_1 = 36 \longrightarrow (1)$$



$\Sigma A\bar{x} = 0$

$\frac{1}{2} \times R_1 L \times (L/3) - M_1 L (L/2) - M' b (b/2) = 0$

② $85.3 R_1 - 32 M_1 = 54$

① $\times 4$ $128 R_1 - 32 M_1 = 144$

$-42.7 R_1 = -90$

$R_1 = 2.10$ $\quad R_2 = -2.10$

$32 R_1 - 8 M_1 = 36$

$8 M_1 = 32 R_1 - 36$

$= 67.4 - 36 = 31.44$

$M_1 = 3.93$

Taking moment about A

$\Rightarrow - R_2 \times L - M_1 + M_2 - M' = 0$

$\Rightarrow - (2.10) \times 8 - 3.93 + M_2 - 12 = 0$

$\Rightarrow 16.8 - 3.93 - 12 + M_2 = 0$

$M_2 = 0.87$ KNm

$y_c = \Sigma A x = \frac{1}{2} \times a_1 \times R_1 (a)(a/3) - M_1 a_1 \times a/2$

$= \frac{1}{2} \times 5 \times 2.1 \times 5 \times 5/3 - 3.93 \times 5 \times \frac{5}{2}$

$= 5.22$

5. Determine the find end moments and reactions for a fixed end beam loaded by a couple a 8KNm acting at the centre of the beam also find the deflection at the point c Where the couple acts the span of the beam is 17m.

$\Sigma A = 0$ the $a = b$

$= \frac{1}{2} R_1 L \times L - M_1 \times L - M' \times b$

$= \frac{1}{2} \times R_1 \times 17 \times 17 - M_1 \times 17 - 8 \times 8.5$

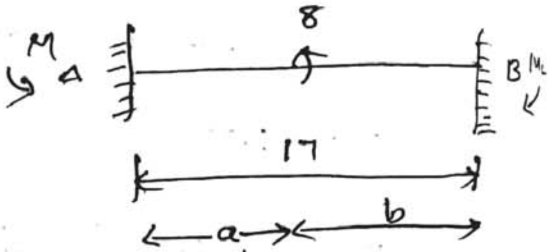
$= 144.5 R_1 - 17 M_1 = 68 \rightarrow (1)$

$\Sigma A\bar{x} = 0$

$\frac{1}{2} R_1 L (L/3) L - M_1 L L/2 - M' b (b/2) = 0$

$= \frac{1}{2} \times R_1 \times 17 \times 17 \times 17/3 - M_1 \times 17 \times 17/2 - \frac{8.805 \times 8.5}{2} = 0$

$= 818.8 R_1 - 144.5 M_1 = 289 \rightarrow (2)$



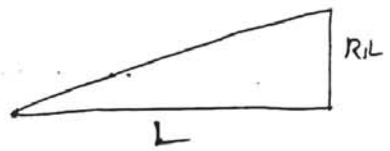
$R_1 = 0.706 KN$

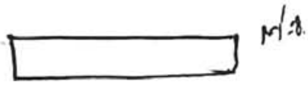
$M_1 = 2 KNm$

$R_2 = -0.706 KN$

$M_2 = -1.985 KNm$

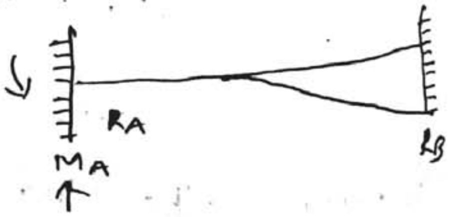
$y_c = 0$ $[\because a = b]$



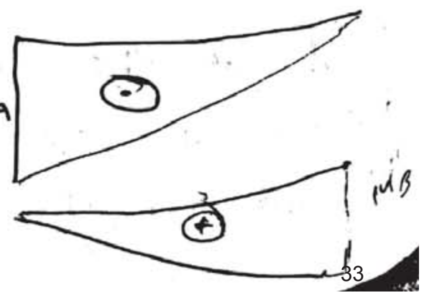


Effect of sinking of supports

→ In simply supported beams the effect of sinking of support will cause no effect in B.M and S.F in the beam.



→ In fixed beams one support settles or rotates, moments are develop at the supports



→ slopes at the points A and B equal to zero

$\frac{M}{EI}$ diagram $= 0$.

Because of this reason the fixed end moments at supports is equal and having oppasite signs

i.e $M_A = -M_B$

$$\boxed{-\Delta = \frac{A\bar{x}}{EI}}$$

$$-\Delta = \frac{M_A \times \frac{L}{2} + \frac{2}{3}L + M_B \times \frac{L}{2} \times \frac{L}{3}}{EI}$$

$$-EI\Delta = \frac{M_A L^2}{3} + \frac{M_B L^2}{6}$$

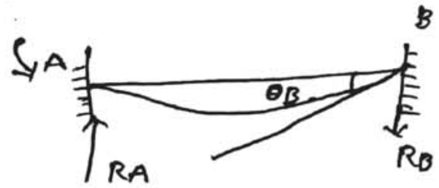
$$-\frac{6EI\Delta}{L^2} = 2M_A + M_B$$

$$M_A = -\frac{6EI\Delta}{L^2} \quad [\because M_A = -M_B]$$

$$M_B = \frac{6EI\Delta}{L^2}$$

Effect of rotation of a support :-

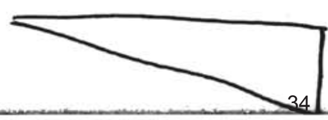
Let rotatanid of support be at B anticlock wise as shown in fig. the rotate b/w the two tangents A and B will be θ_B. let M_A and M_B be the fixed moments at two supports



$$\theta_B = \frac{A}{EI}$$

MA

$$\theta_B = \frac{M_A \times l/2 + M_B \times l/2}{EI}$$



MB.

$$\Rightarrow \quad M_A + M_B = \frac{2EI\theta_B}{l}$$

$$\Rightarrow \quad \frac{M_A \times l/2 \times 2/3\,l + M_B \times l/2 \times 2/3}{EI} = 0$$

$$\Rightarrow \quad 2M_A + M_B = 0$$

$$M_A = \frac{2EI\theta_B}{l}$$

$$M_B = \frac{4EI\theta_B}{l}$$

1. A beam AB of span 4m fixed at A end B carries a one of 1500N/m the support B sincks by 1cm find the fixed end moments and draw BM diagram for the beam Modulus of elasticity $E = 2 \times 10^5\ N/mm^2$ moment of inertia $I = 8000cm^4$

$$M_A = M_B = -\frac{wl^2}{12} = -2000Nm$$

Due to settelment of support B

$$M_A = -M_B = \frac{6EI\delta}{l^2}$$

$$= \frac{6 \times 2 \times 10^5 \times 8000 \times 10^4 \times 10}{(4000)^2}$$

$$= 6 \times 10^7\ Nmm$$